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DUAL EXCHANGE RATE SYSTEMS

A Thesis presented for the Degree of M.Phil. in Economics

by

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This work could not have been done without the help of David Currie, who supervised it; The Foreign and Commonwealth Office, who subsidised it; and Sue, who tolerated and typed it.

DUAL EXCHANGE RATE SYSTEMS: ABSTRACT

This thesis is in two chapters. The first surveys the existing literature on 'dual exchange rate systems' - where current and capital transactions are channeled through separate foreign exchange markets. The early, comparative static, models concentrate on whether the instruments of demand management will work, and conclude that under dual regimes, unlike with fixed or floating unified rates, both fiscal and monetary policy can be effective. Later, dynamic, models focus on the impact of exogenous capital flows (usually represented by external interest rate movements) on economies with dual systems. The process of expectations formation proves crucial - so rational expectations appear naturally in a crop of papers whose shared conclusion is that a dual system entirely insulates the domestic economy from the effects of such flows. Finally, there have been studies of dual systems in practice. Their chief conclusion is that the benefits have been more questionable than theory would suggest.

The rational expectations model of a dual exchange market described in Chapter II goes beyond previous models in a number of ways. It permits precise comparison of the insulation properties of dual rates with those of other regimes. It allows for Government intervention between the two exchange markets under a dual regime. And it analyses the case where only imperfect information is available on the movements of external variables. The chief results are that for an open

economy producing a differentiated product a dual system is likely to be more effective than either fixed or floating rates in insulating domestic activity from external interest rate movements (although perfect insulation is not achieved), and no less effective in insulating it from external price changes. This remains true when only imperfect information is available and also (although diminishingly so) when the Government intervenes between the exchange markets for capital and current transactions.

CHAPTER ONE: A SURVEY OF THE LITERATURE

I Introduction

At about the time of the 1971 changes in the Foreign Exchange Markets there was a burst of international enthusiasm for foreign exchange régimes in which current and capital transactions are channeled through separate markets, often at different exchange rates. At the end of 1970 only 9 countries maintained such systems. By the start of 1973 26 countries (out of 127 surveyed by the IMF) maintained some form of dual market. This group included France and the French Operations Account Countries, Italy, the Belgium/Luxembourg Economic Union, the Netherlands and South Africa. Interest waned however almost as rapidly as it had arisen. In the course of 1974 France (and the Operations Account Countries), Italy and the Netherlands all abolished their dual markets. Others have lasted longer. South Africa's 'financial rand' only finally disappeared in 1983, and the Belgium/Luxembourg economic union continues to maintain a dual market (there are also dual markets of a rather different kind still functioning in a number of third world countries, notably Egypt and Argentina - we do not consider these here).

Academic economists batted early onto the phenomenon of dual markets. Fleming (1971) was probably the first to look at the arrangements which would be required for a viable dual market system, and their likely consequences. He was closely followed by Salin (1971) and Barattieri & Regazzi (1971) and reverted to the subject himself in 1974 with his comparison

of the effectiveness of dual markets with other forms of control on capital movements. There was then a number of, in general more mathematical studies looking at the effectiveness of the traditional tools of economic management under dual exchange rate régimes; including Argy & Porter (1972), Decaluwe (1974), Swoboda (1974), Foll (1976), Dornbusch (1976), Decaluwe & Steinherr (1976) and Marion (1981). A number of these authors emphasise the key role played by the formation of exchange rate expectations in any analysis of dual markets, while introducing rather ad hoc hypotheses for this into their models. The lacuna is partially filled by Flood (1978) who constructs the first model in which expectations are formed rationally. This is then followed by the more comprehensive and persuasive models of Flood & Marion (1982) and Cumby (1984).

Meanwhile a number of other authors have looked at how dual markets have worked out in practice. Lanyi (1975) draws general conclusions from the Belgian, French and Italian experience, as well as usefully summarising the rules that operated in those three markets. Otherwise, the Belgian experience has monopolised academic attention, being examined in Talent (1970), De Looper & Mountford (1973) and (most extensively) in Decaluwe and Steinherr (1976) to which De Grauwe (1976) is a reply.

This Chapter follows the pattern that the literature has taken. Section II introduces the basic definition of a dual market and looks at some immediate consequences of that definition; the reasons for adopting such a system, the institutional arrangements required and the problems which will

obviously arise. Section III looks at the work that has been done on the effectiveness of fiscal and monetary policy under dual exchange rate régimes. Section IV looks at the questions of expectations formation in, and stability of, dual markets. Finally, Section V summarises the conclusions that have been drawn from the operation of dual markets in practice.

II Definitions and First Consequences

It is widely agreed that one of the chief factors that undermined the Bretton-Woods system of fixed exchange rates was the growth and growing volatility of international capital movements. The idea of a dual system of exchange rates (henceforth dual market) was proposed as a means by which countries could prevent such flows from affecting the more normal run of their overseas transactions. In a dual market capital and current account transactions are conducted through separate foreign exchange markets. Terminology varies. Henceforth in this survey the market through which current account items are transacted will be referred to as the Commercial market, that for capital items the financial market. Since the primary aim of establishing such a system is to accommodate speculative capital flows while insulating the commercial exchange rate from them, that rate has generally, both in practice and in theoretical studies, been kept fixed while the financial rate has floated. This assumption will be retained throughout, except where it is specifically stated otherwise.

Fleming (1971) looks in detail at the controls necessary to separate the financial and the commercial markets. The Government must establish extensive controls over at least one of the two categories of foreign exchange transaction

carried out by domestic residents (usually the current items). They must also establish controls over the domestic currency transactions of non-residents - which has usually been done by separating non-resident accounts into two types, one for current transactions and one for capital transactions.

Assuming that the establishment of such controls effectively separates the commercial and financial markets, what is the effect on the international capital flows which gave rise to the idea in the first place? All authors note that the answer depends on the detail of the way in which the market is managed. Three possible effects are as follows:-

(a) Capital Gains: Decisions about inflows of foreign capital will be affected by expectations about the future movement of the financial exchange rate, and the likely consequent gain or loss when the foreign investor comes to withdraw his capital. To the extent that the financial rate is expected, in the long run, to remain close to the (fixed) commercial rate this is a stabilising factor - if excess domestic demand for overseas capital causes the financial rate to rise to a premium over the commercial rate then (self-fulfilling) expectations of a subsequent fall will lead to a compensating fall in demand for overseas capital.

(b) Rates of Return: This depends on which market is used for interest payments on overseas holdings of domestic capital and domestic holdings of foreign capital. There are two possibilities - the Belgian style of dual market where interest payments are converted at the

financial rate, and the Italian style where they are converted at the commercial rate (the terminology is mine, the distinction is implicit in a lot of the literature without being systematically explored - in general the literature has concentrated on Italian style dual market systems while the Belgian style has been more prevalent in practice). In an Italian style dual market system the return on domestic capital held by foreigners can diverge from the domestic interest rate. In fact if the nominal domestic interest rate is r , the financial exchange rate is e_f (units of domestic currency per unit of foreign currency) and the commercial exchange rate is e_c , then it is easy to see that the return earned by a unit of foreign currency invested domestically (through the financial market with the interest repatriated through the commercial market) is;

$$\frac{e_f \cdot r}{e_c}$$

Thus as the financial rate varies so the rate of return on, and hence the demand for, domestic capital by foreigners also varies. Again this effect looks stabilising - inflows of foreign capital cause the financial rate to appreciate (ie. e_f to fall) and so diminish the rate of return on such inflows. (It should be added

however that Decaluwe and Steinherr 1976b have demonstrated that in certain circumstances a Belgian style dual market can be stable when an Italian style is not - see below)

(c) Intervention Policy: A crucial determinant of the extent of capital inflows/outflows is the intervention policy of the authorities in the financial exchange market (by assumption their intervention policy in the commercial market is dictated by the need to keep the rate there fixed). This was first pointed out by Fleming (1971), although our terminology is that of Lanyi (1975). He makes the distinction between a policy of non-intervention - in which the financial rate floats freely - and a neutral intervention policy (first suggested by Barattieri and Ragazzi 1971) in which the monetary authority sells (buys) foreign exchange in the financial exchange market equal to the net increase (loss) in official reserves resulting from a current account surplus (deficit). Lanyi also emphasises that neutral intervention has the advantage of affecting the very considerable leakages (eg "Leads and lags" see below) which must take place between the two markets.

All those authors who have examined dual markets run under a policy of non-intervention have pointed out that in such circumstances all net capital movements into or out of the country are choked off. For if, for example, a domestic resident wants to buy internationally traded securities he must either buy them from another domestic resident (no net capital flow) or purchase them from a foreigner. In the latter

case he must first buy foreign exchange through the financial market. But since that market is always balanced and, by assumption, there is no external intervention, he can only buy such foreign exchange if someone else is prepared to sell - i.e. another domestic resident is selling foreign securities, or a non-resident wishes to buy domestic securities - in either case the net capital movement is nil.

Under a neutral intervention policy on the other hand there are net capital flows to the extent of the official intervention, but there is also overall balance of payments equilibrium - with the imbalance on current account precisely offset by an equal and opposite imbalance on capital account. It should be added, however, that (as pointed out by Fleming (1971)) in these circumstances the domestic currency accounts of the monetary authority will not necessarily be in equilibrium. Indeed (retaining the notation introduced above) if $e_f \neq e_c$ and the current account balance (in foreign currency terms) is ϵ then in order to run a neutral intervention policy the monetary authority must add to the domestic money supply at a rate $(e_c - e_f) \cdot \epsilon$.

This can of course be sterilised but it does mean that in such circumstances the domestic economy is in a state of quasi-equilibrium rather than true equilibrium (Boll (1976) has taken this analysis a little further - to point out that, other factors remaining constant, the financial exchange rate under a neutral intervention policy is explosively unstable under slight exogenous variations of the domestic money supply - for if starting from equilibrium the money supply rises, activity rises and the current account goes into deficit,

domestic interest rates fall, there is reduced overseas demand for domestic capital, and rises, by the mechanism described above, money supply rises again and so on).

These three mechanisms by which the introduction of a dual market can affect capital movements all depend on the ability of the authorities to set up a system of regulation effectively separating capital and current flows. But, as a number of authors (Fleming 1971, 1974, Lanyi 1975) have pointed out, it is impossible for practical reasons entirely to disentangle the current and capital accounts. Firstly there are a number of capital transactions which, in normal circumstances, it is impossible to divert out of the commercial market - notably changes in non-residents "commercial account" balances in domestic currency (an important source of currency speculation in the past) and "leads and lags" on trade payments - all of which could be affected if, say, a devaluation of the commercial rate were expected. Secondly there are certain types of international transactions which, while technically current account items, can easily be used for the acquisition of foreign assets - these include remittances of savings by foreign workers, tourist expenditure and other cash transactions. Those transactions have therefore normally been channelled through the financial market. A third link may be created through the operation of forward markets - even when there are separate "financial" and "commercial" forward markets they can create indirect links between the two markets because e.g. if the financial forward rate is at a premium this could create

expectations of a revaluation of the commercial rate with consequent speculative pressure. Finally Lanyi extensively chronicles the possibilities for false invoicing and other methods of illegally channelling e.g. current payments through the financial market. The key point here is that the incentive for, and so the extent of, this kind of evasion will increase rapidly as the gap between the financial and the commercial rates widen.

The existence of these limitations on the ability of the authorities to separate current and capital transactions raises the question of how the use of dual markets compares with other means of controlling autonomous capital flows. Fleming (1974) looks at a range of such alternatives and reaches the following conclusions;

quantitative restrictions on capital movements: the dual market system has the advantage over these of being automatic and of allocating funds efficiently among different types of capital flow (Fleming also claims that they are cheaper to administer, but Lanyi points out that this is not necessarily the case if the costs of double bookkeeping for overseas accounts is taken into consideration). However it is more likely to give rise to disequilibrating exchange rate speculation e.g. if the market takes a wide gap between the two rates as a signal that the commercial rate needs to be devalued.

forward exchange market intervention: forward market intervention can affect leads and lags, which dual markets cannot. On the other hand forward market intervention can be expensive and chiefly affects only those foreign exchange transactions which are covered in the forward market. Fleming concludes that this is a useful adjunct to the use of dual markets.

fiscal and monetary policy: the use of a dual market is much more rapidly responsive to conditions in the market place and is less likely to interfere with the achievement of other macroeconomic objectives.

fixed exchange rate, borrowing abroad where necessary: a dual market relieves official reserves from the effect of volatile capital movements (with their monetary consequences) - but distorts economic activity more and (under a policy of neutral intervention) may conceal a fundamental imbalance in the commercial market larger than would be possible with fixed rates.

floating exchange rates: dual rates are more distortive, and less able to respond to deep underlying imbalances in the current account, but do provide an environment of greater exchange rate stability for foreign trade provided the commercial rate is set at a realistic level.

Fleming concludes that dual rates have important advantages over most of the other major methods of controlling autonomous capital flows. In view of the danger, however, that they may conceal persistent disequilibria he suggests that a hybrid system may be worth examination - in effect this would be a dual market run under a neutral intervention policy in which the commercial exchange rate instead of being fixed would be subject to a managed float in line with long run trends in the current account. This idea has received little follow up in the literature. the only two references to such a system being a brief comment in Lanyi (who notes that the effectiveness of a hybrid market are bound to depend on the intervention policy adopted, and that it exposes the commercial exchange rate to speculative pressures) and its inclusion as one of the

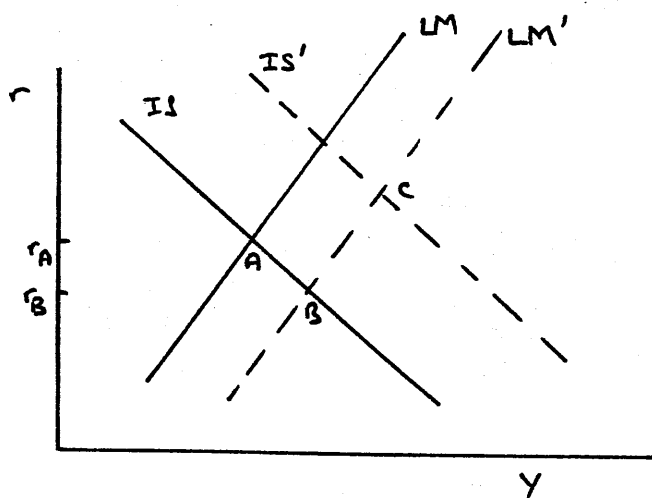
exchange rate systems examined in Flood and Marion (1982). Nor has it been employed, in its pure form, in practice, although the current Belgian system in which the fixed level of the "commercial" franc in the EMS is subject to frequent small readjustments is quite close to it.

III Dual Rates and the Instruments of Economic Policy

Mundell and Fleming have demonstrated that for a small open economy under perfect capital mobility the effectiveness of fiscal and monetary policy for domestic activity depends on the exchange rate regime. Under fixed rates fiscal policy works and monetary policy does not, while under flexible rates the reverse is true.

Since a dual rate system has some of the qualities of fixed rates, and some of floating, it is plainly of interest to consider the effects of fiscal and monetary policy under them. A number of authors (in particular Boll, 1976) have looked at this question. For example, using the standard IS/LM diagram shown, consider the effect, under a dual currency system with non-intervention, of a rise in the money supply from LM to LM'.

The economy will begin to move from point A to point B. If there were a unified fixed exchange rate, the outflow of capital provoked by the incipient fall in domestic interest rates from r_A to r_B



would reverse the rise in money supply and take the economy back to point A. Since, however, as noted above, there can be

no net capital movement in a dual rate system with non-intervention the effect is instead of greater demand for foreign exchange on the financial market and a rise in e_f . The consequence of this is different according to whether we have a Belgian type or Italian type dual market;

(i) in an Italian type market a rise in e_f implies a rise in domestic interest rates as seen by outsiders (and a fall in overseas interest rates as seen by domestic residents). So the economy can move to point B while the financial rate depreciates to the point where $e_f/e_c \cdot r_B = r_A$ and this is a quasi-equilibrium (the current account is of course in deficit). It follows that under an Italian type dual market with non-intervention monetary policy is effective in adjusting the level of domestic activity (Swoboda, 1974 emphasises that this is not a sustainable equilibrium due to the imbalance on the current account - we will return to this point below) - it is also worth noting that under flexible rates the economy would move to point C - a greater effect on income than under dual rates.

(ii) in a Belgian type dual market on the other hand the interest rate effect does not operate. Hence within this model there is no way of eliminating the surplus demand for foreign exchange on the financial market. e_f rises perpetually. This evident inadequacy of the IS/LM framework to describe the workings of a Belgian type dual market leads to two interesting conclusions:

(a) the decision in a dual market system as to which market interest rates should be paid through is more significant than it may initially appear. More precisely, an Italian

type decision to channel interest payments through the commercial market establishes a direct link between monetary conditions and the overseas account which otherwise does not exist.

(b) quite clearly in a Belgian type market ~~ef~~ cannot respond to a rise in money supply by rising perpetually - and the best hypothesis to eliminate this possibility (in a perfect theoretical world where leakage between current and capital transactions can be prevented) must be on the expectations generated by a steadily growing gap between the financial and commercial rates. This is a first indication of the importance of expectations in the working of a dual market system, to which we will return below.

Confining our attention for the remainder of this section to Italian type markets it is possible to demonstrate by similar analysis to that above that fiscal policy is also effective in adjusting the level of domestic activity in a dual market with non-intervention (the only difference being that while monetary expansion leads to a depreciation of the financial rate, fiscal expansion leads to an appreciation). The same kind of analysis shows that the same conclusions apply to dual markets under neutral intervention - an unsurprising result given that, from the point of view of this model, the only difference between non-intervention and neutral intervention is that in the former there is no net capital movement, and in the latter there is just enough to offset any change in the state of the current account. (Since these results are not entirely clear in Boll they are given in Appendix I.)

Boll goes on to use the same type of analysis to consider the possibility of achieving under four different exchange rate regimes (fixed rate, floating rate, dual with non-intervention and dual with neutral intervention) a variety of macroeconomic targets; full employment (i.e. $Y=Y_0$), external balance, and optimal growth (i.e. $r=r_0$). Clearly the last objective is not meaningful (at least for unified exchange rate systems) under perfect capital mobility, a hypothesis which he therefore drops in favour of high capital mobility (i.e. the external balance curve at equilibrium and the prevailing exchange rate(s) has a lower slope than the LM curve;

$$\left. \frac{\partial r}{\partial Y} \right|_{\substack{B=B_0 \\ e=e_0}} < \left. \frac{\partial r}{\partial Y} \right|_{\substack{M=M_0 \\ e=e_0}}$$

On this assumption both fiscal and monetary policy are effective. (to varying extents) under all four exchange rate régimes.- It follows rather straightforwardly that under those two régimes where external balance is guaranteed (floating rates and dual markets with neutral intervention) use of fiscal and monetary policy is enough to bring the economy to quasi-equilibrium at a point where all three objectives are met. With the other two regimes however (fixed rates and dual markets with non-intervention) the authorities have only two instruments with which to pursue three objectives and thus will not be able to achieve them all, except by chance.

Decaluwe and Steinherr (1976a) also consider the question of whether the introduction of dual markets might help the policymaker simultaneously to achieve several macro-economic objectives. In particular they point out that whereas a deficit

in the capital account does not effect domestic wealth, a deficit in the current account does. There may be circumstances therefore, in which the authorities will wish to set separate targets for net international indebtedness and the level of international reserves. Would the establishment of a dual market be an appropriate mechanism for this? To help answer this question they set up an elaborate portfolio balance model of a small economy with an Italian style dual exchange market. operating under non-intervention. In order to complete their model they have to introduce the (appealing but arbitrary) hypothesis that the expected future value of the two exchange rates appreciates with the balance of trade, and can depend also on the current gap between the two rates (although speculative movements of the trade balance in response to these expectations i.e. "leads and Lags" are excluded). Their chief conclusions are as follows:-

i) In the short run, on a comparative static analysis, the introduction of a dual exchange market does make it possible for the authorities to pursue separate targets for net international indebtedness and the level of international reserves. Furthermore, as under IS/LM analysis, both fiscal and monetary policy are effective economic instruments even when international capital is highly mobile.

ii) In the long run equilibrium, and contrary to Swoboda's conclusion quoted above, monetary policy can effectively influence long run income and the level of domestic interest rates, provided that expectations

of the future commercial and financial rates are formed independently of one another (i.e. dependence on the 'gap' is zero). This is because Swoboda's IS/LM analysis does not include interest payments on foreign debt.

As noted above if it is possible in long run equilibrium to have $e_f \neq e_c$ (which can be done when their expected values are independent of the gap between them) then it is possible to have a domestic interest rate less than/greater than the international interest rate, and so to run a trade account surplus/deficit while still remaining in current account equilibrium.

Marion (1981) pursues the portfolio balance approach but reworks the Decaluwe and Steinherr model so as to establish a more vigorous distinction between stock and flow items. Assuming static expectations she then uses this model to examine the effectiveness of a dual system in insulating the economy from overseas disturbances. Her chief conclusion, to which we will return below, is that (assuming overseas bonds are of a certain type) dual rates, unlike fixed and flexible rates, can completely insulate the domestic economy from variations in the overseas interest rate (such movement, being sterilised by compensating movements in the financial exchange rate - which has no effect on the real economy due to the rigorous segregation of financial and commodity sectors in this model).

Aizenmann (1983) uses a variant of the Marion portfolio balance model to compare the short term effects of domestic

monetary policy and devaluation of the (commercial) exchange rate under fixed rates and Italian type dual rates. He shows that under dual rates the impacts of these policy actions on domestic interest rates are sharper, but more transient, than the same changes under unified fixed rates. He also demonstrates that these results hold under both static and rational expectations. Much of his argument, in the static expectations case, is capable of substantial simplification (see appendix II). Obstfeld (1984) takes the analysis of the effects of devaluation on a dual rate economy considerably further, showing in particular that inclusion of the assumption that central bank foreign reserves earn interest results in devaluation becoming non-neutral - i.e. having a real impact on the long term equilibrium of the economy.

IV Expectations and Stability

As remarked above, the IS/LM analysis of Boll is inadequate to study the effects of policy changes on Belgian style dual exchange markets because in such markets there is no direct link between monetary conditions and the balance of payments. Some new hypothesis is needed to re-establish that link, and the most appropriate such hypothesis will probably concern expected future values of the financial exchange rate.

Such a model was set up by Argy and Porter (1972). They examine the response of a small open economy under a variety of exchange rate regimes and perfect capital mobility to a number of exogenous disturbances, including variations

in fiscal and monetary policy, and in the external interest rate. The model is a straightforward comparative static model incorporating (in the incarnation which interests us here) a Belgian style dual exchange market without Government intervention. The key assumptions are;

i) the forward (= expected) rate on the financial market is given by;

$$e_f(t) \text{ (forward)} = e_f(t) + \lambda (e_f(t) - e_f(t-1))$$

where λ is a constant reflecting the elasticity of expectations. $-1 \leq \lambda \leq 1$

ii) demand for net overseas debt (the level of which must remain constant under a non-intervention policy) depends upon the expected return on such debt;

$$B = B(r-r^* + \frac{e_f(\text{forward}) - e_f}{e_f}) \quad \text{where } r^* = \text{international interest rate}$$

Running this model gives rise to the following interesting results;

a) when $\lambda = 0$ (i.e. expectations are static) we are back in the situation of the Boll model where the money market is overdefined, the domestic interest rate being fixed by international capital mobility, and domestic supply of money being held fixed by the impossibility of net capital flow. As with Boll the model does not work in this case.

b) if $\lambda \neq 0$ then income and interest rates are completely independent of the precise value given to λ (i.e., on a comparative static analysis, the existence of non-static expectations for the financial rate are a necessary part of the model, but the precise nature

of those expectations do not affect the results that the model yields).

c) in a Belgian type dual market with expectations monetary policy is effective in adjusting national income (e.g. a rise in money supply will lead to a fall in the domestic interest rate and expansion of output. The resulting imbalance of demand in the financial market will be accommodated by a movement of the spot and forward financial rates in such a way that

$$r - r^* + \frac{e_f(\text{forward}) - e_t}{e_f} = 0$$

and hence the financial market remains balanced. The only effect of λ taking different values is that the spot and forward rates move by varying amounts) - although, as with Italian type markets, the effect on income is less than would be produced by the same monetary expansion under unified flexible exchange rates.

d) an analogous argument demonstrates that under the same assumptions fiscal policy is also effective in adjusting national income.

e) Argy and Porter also consider the effect of an exogenous shift in international interest rates, r^* . r and y are insulated from this and remain unchanged, the pressure on the capital market being again absorbed by compensating movements in the spot and forward financial rate. This contrasts quite sharply with the behaviour of the same model under fixed or flexible unified rates, where domestic income is affected by

exogenous shifts in r^* , and demonstrates that, in this model at least, the existence of a dual market is effective protection against international capital movements.

Flood (1978) supports the view originally expressed by Fleming (1971) and subsequently supported in the work of Decaluwe and Steinherr, Dorbusch and Argy and Porter that the process of expectations formation is central to any analysis of how dual exchange markets work. He points out however that the forms of expectation formation used by these previous authors has tended to be ad hoc - and may therefore be assuming away an important part of the story of dual markets. He therefore sets up a monetary model of a small fully employed open economy under perfect capital mobility with an Italian type dual market operating with no government intervention. Expectations are introduced, as in Argy and Porter, through the expected rate of return on internationally traded securities;

$$\rho(t) = \frac{e_c}{e_f(t)} \cdot r^*(t) + \frac{e_f(t+1)^{\text{expected}} - e_f(t)}{e_f(t)}$$

and the model is driven by independent stochastic variations of mean 0 in domestic credit and the international interest rate, r^* . Finally, the distinctive feature of the model, expectations of the financial exchange rate are assumed to be formed rationally in the sense of Muth - the expected value of $e_f(t+1)$ is its mathematical expectation at time t .

The most interesting result to emerge from Flood's analysis is that, contrary to the assumption made by Decaluwe and Steinherr (1976a) the expected financial exchange rate depreciates as the current account improves. On inspection the reason is simple - a current account surplus leads to an expanding money supply, and hence the opportunity cost of holding money (i.e. $\rho(t)$) must fall - so e_f must rise.

Flood and Marion (1982) take this approach considerably further by completely rebuilding the Argy-Porter model in a rational expectations mould. In effect their new model is a variant of the Dornbusch and Fischer approach in which the exchange rate is set by capital movements, and balance on the current account is achieved, over time, by the effect of asset movements working through the wealth effect. As in Flood's earlier model, expectations play a role through the expected rate of return on domestically held international securities and those expectations are again assumed rational. This sophisticated apparatus (which is not solved in detail) is designed, and used, to examine the question of what proportion of wage contracts should, optimally, be indexed against price movements under four different exchange rate systems: unified fixed, unified floating, what we have called Italian dual, and a fourth systems which is our Italian dual system in which the commercial rate too is allowed to float. The central conclusion of the paper is that the optimal level of wage indexation (and hence the central structural equations of the system) will depend upon which exchange rate system is adopted and, in particular, that 100% indexation is optimal for

(and only for) those two exchange rate systems where the exchange rate for commercial transactions is fixed. Moreover if 100% indexation is adopted under such systems then it completely insulates the level of domestic production from overseas disturbances (i.e. rediscovering Marion's 1981 result from a different direction). In a final section the authors show that their results do not extend to the case where the economy has influence over the price of its own products in export markets.

Cumby (1984) goes some way to fill this lacuna by adapting the Dornbusch/Fischer approach, with perfect foresight and a dual exchange rate system, to an economy possessing market power. His chief conclusions (both of which will be re-examined in the context of the more precise analysis provided by the model in Chapter II below) are that such a system always exhibits the saddle point stability necessary for rational expectations models, and (again) that the dual exchange rate system completely insulates the domestic economy from variations in the overseas interest rate. He also shows that under such a régime the monetary authority is able (through open market operations) to exert long term control over the domestic money supply and interest rate, but not to sterilise the long term effects of variation in overseas price levels.

Finally for this section we look briefly at the only model hitherto which looks at the effect of government intervention. This is in Decaluwe and Steinherr (1976b). They assume that a proportion $(1-\alpha)$ of the current account deficit is financed from the financial market (i.e. $\alpha = 1$

means non-intervention, $\alpha = 0$ means neutral intervention).

The rates of change of the domestic interest rate and financial exchange rate are assumed to be proportional to the excess demand in the money and traded capital markets respectively, and the expected future value of the financial exchange rate (upon which net demand for capital inflows/outflows is assumed to depend) is assumed to depend only on the domestic interest rate (overseas parameters, including the overseas interest rate being assumed constant) and to appreciate as the domestic interest rate rises.

) From this rather ad hoc, if plausible, set of hypotheses Decaluwe and Steinherr derive the following conclusions;

a) If the market is run under Italian type rules then there is a possibility that it is unstable, depending upon the values given to the various parameters. Assuming that interest rates and capital flows respond rapidly to imbalances in their respective markets, however, it is clear that α becomes a key parameter. More precisely the higher the level of official intervention in the financial market the more likely it is to be stable. The reason for this is that, in the absence of intervention, a current account surplus increases domestic money supply. This exerts downward pressure on the domestic interest rate and creates excess demand for foreign exchange in the financial market and a depreciation of the financial rate. Furthermore the fall in the domestic interest rate improves the current account further, by reducing overseas interest payments, and in certain circumstances the process can become explosive.

With official intervention, however, part of the current account surplus is sold on the financial market, so the money supply rises less, interest rates fall less and an explosion is less likely.

b) The above conclusions are still valid if one introduces the additional hypothesis that capital flows are negatively affected by expectations of a fall in the commercial rate if the discount of the financial rate against the commercial rate widens (i.e. the "leads and lags" effect).

c) If the same model is run under Belgian type rules, it is always stable, whatever the level of Government intervention. This is because the interest rate link between monetary conditions and the current account is broken and hence the sort of instability described above is not possible. (The model in Chapter II below shows that the introduction of rational expectations leads to entirely different conclusions about the effects on stability of varying levels of Government intervention.)

V Dual Markets in Practice

The most successful, or at least most enduring, dual exchange market has been that in existence in the Belgium/Luxembourg Exchange Union since 1951. It has consequently also received the lion's share of academic attention (and, conversely, a disproportionately large number of the academics who have studied two tier markets have been Belgians). In particular, undoubtedly the fullest and most thorough study of a dual exchange market operating in practice is that by Decaluwe and Steinherr (1976b) on the Belgian experience in the years 1961-71. The paper concentrates on three questions:

a) granted that there is bound to be same leakage between the two markets, was it nevertheless possible to establish an effective split between capital and current flows?

b) if so, did this have the desired effect of controlling exogenous capital movements?

c) was monetary policy, as predicted by theory, an effective macro-economic instrument under a dual exchange rate régime?

Decaluwe and Steinherr open their paper by summarising the structure of the Belgian dual exchange market in the period 1961-71. It was a Belgian-type market (i.e. interest payable through the financial market) with the additional peculiarity that capital outflows (but not inflows) could take place through the commercial market - the immediate consequence of which is that the financial rate can be at a discount, but not a premium, to the commercial rate. The chief features of the markets operation in the period under consideration were as follows;

i) throughout most of the period there was very limited divergence of the two exchange rates (usually less than 1%), partly because of world financial stability and partly because of countercyclical purchases of foreign debt by the Belgian public sector when the financial rate was at a discount.

ii) there was a firm correlation throughout the period between the strength of the current account and the level of the financial rate. As the current account went into deficit so the financial rate fell i.e. "leads and lags" and other linkages between the current and capital accounts were enough

to ensure that they weakened or strengthened together.

iii) following from (ii) there was also a strong correlation between the differential between domestic and international interest rates and the exchange rate differential.

iv) since this was a Belgian type system, the major influence on capital flows was the expected movement of the financial rate. The evidence of the period is that in 1966, 1968 and 1969 major capital outflows were discouraged by a falling financial rate (on those three occasions the exchange rate differential opened to a point wider than the interest rate differential).

v) throughout the period up to 1970 the monetary authorities pursued a neutral intervention policy, selling on the financial market the foreign exchange resulting from the current account surplus Belgium had in those years.

vi) however, at the time of the exchange rate crisis of 1971, when the current account went into deficit and there was strong speculative pressure on the financial franc the authorities felt unable to finance the current account by intervening against the financial franc (as they would have had to do).

At the same time there was considerable arbitrage within the domestic banking system (which at that time was inadequately regulated) tending to further weaken the financial franc, as well as considerable extra "leakage" between the two accounts. As a result the authorities were finally forced to impose quantitative controls on movements of capital for a short time.

vii) Decaluwe and Steinherr suggest that the key difference between 1971 and the period that preceded it is that in 1971 the authorities were confronted with outflows on both capital

and current accounts, whereas before the two accounts had been pushing in different directions. A dual market system may be more effective in curbing sudden capital movements in the first case than in the second.

viii) Decaluwe and Steinherr then construct a model of the financial and exchange markets incorporating the linkages noted above, and estimate the various parameters on the basis of the 1961-71 experience. They get quite a good fit, and arrive at the following answers to the three questions they raised at the start of their analysis;

- (a) In normal circumstances, the dual market did achieve an effective separation between capital and current payments. When the exchange rate differential became **sizeable** however leakage and speculation transactions in the commercial market grew and it became increasingly difficult to separate the two markets.
- (b) Up to 1971 the existence of the dual market did reduce the effect of exogenous capital movements.
- (c) Monetary policy was apparently more effective under the dual market regime than might otherwise have been expected. Only about 57% of changes in the domestic money stock were offset by reserve movements.

De Grauwe (1976) criticises most of Decaluwe and Steinherr's conclusions. He points out that the Belgian experience in 1971 and subsequently shows that the dual market cannot control major speculative movements, and so the system is most effective when least needed. The proceeds from the effective 'tax' imposed on capital outflows by the dual market system went to

those who arbitrated illegally between the two markets. The "offset coefficient" of 57%, demonstrating the relative autonomy of the money supply in Belgium did no such thing. The corresponding coefficient for Holland, under unified fixed rates, was 59%. And comparing movements of Belgian and Dutch interest rates with Eurodollar rates suggested that all tended to move together - again casting doubts on claims of greater Belgian monetary autonomy. Finally de Grauwe concludes that the Belgian dual market prior to 1971 operated as a mercantilistic device - preventing a current account surplus from being balanced by capital outflows.

Lanyi (1975) asks similar questions to Decaluwe and Steinherr in a more impressionistic way, but on the basis of a wider sample. He looks at the records of the French, Italian and Belgian dual market systems from two points of view;

- (a) how effective have they been in defending the exchange rate against speculative movements of capital;
- and (b) have they contributed to the achievement of other balance of payments objectives e.g. maintenance of a certain level of reserves and a certain capital flow, or maintenance of long term equilibrium.

On (a) his answer is "not very". He notes that if speculative flows are to be contained then the market must be able to maintain wide spreads between the two exchange rates when necessary. In fact spreads have tended to remain below 5% and at times of heavy speculative pressure the authorities have had to impose direct controls on capital movements and/or alter the commercial exchange rate. Lanyi

suggests that the reason why wide spreads cannot be sustained is that they increase the incentive to evade the split between the two markets to the point where the two markets become powerfully linked. But if it is not possible to achieve a wide spread then the prospect of earning significant capital gains by buying bonds through the financial market disappears, and speculative pressures on the financial market tend to spill over to the commercial market as well.

On (b) Lanyi reaches the following conclusions;

- i) If long run balance of payments equilibrium is desired then in the long term, the authorities must follow a neutral intervention policy. In practice, however, experience shows that they have preferred a non-intervention policy (and to that extent dual markets have not received a proper test in reality) - apparently because of fears that if they are seen to intervene in the financial market they may provoke speculation and evasion (especially if e.g. they are forced to intervene the 'wrong' way when the current account is in deficit and the financial rate at a discount).
- ii) Hence as a matter of fact governments have felt the need to keep both markets in rough balance (and the FRG Government decided not to introduce a dual market in 1971 precisely because it knew it could not do this).
- iii) Dual markets should be an effective way of pursuing two separate exchange rate targets provided the emphasis is given to the current account (since the capital account will always be affected by short term reversible movements).

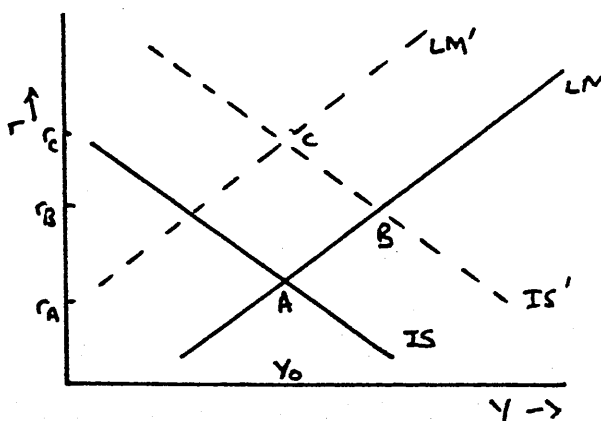
iv) Finally the existence of a dual market does provide an effective system for signalling a basic disequilibrium in the balance of payments (i.e. a persistent divergence in one direction of the financial rate from the commercial rate under neutral intervention)but the existence of such a divergence is likely to be less effective in compelling an exchange rate change than e.g. the steady loss of reserves under fixed rates.

Appendix I

Effects of Fiscal and Monetary Policy under an Italian Type Dual Market

A. Fiscal Policy

(i) non-intervention



We assume perfect capital flow throughout. The state of the current account depends only on Y and is in balance at Y_0 . At the initial equilibrium A: $Y = Y_0, e_f = e_c$ and $r = r^*$ (the international interest rate). Fiscal expansion now takes place moving IS to IS' and the economy to B. r rises to r_B (say) and the rise in interest rates causes the financial rate to rise until

$$e_f/e_c r_B = r^*$$

B is now a quasiequilibrium, with the current account in deficit. In the long term however the fall in reserves will cause the money supply to fall to LM'; and the economy to move to point C with interest rate r_c . The financial rate will continue to appreciate until

$$e_f/e_c r_c = r^*$$

and this is now a long term equilibrium with the current account back in balance. $Y = Y_0$ again, but the interest rate and e_f have changed.

(ii) neutral intervention:

Using the same diagram as above with the same initial equilibrium and the same expansion to IS' the economy moves to B and as before the financial rate appreciates until

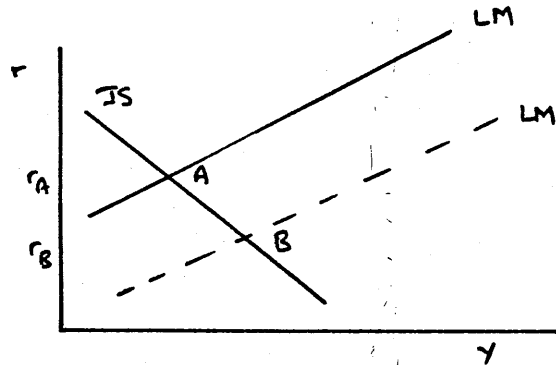
$$e_f/e_c r_B = r^*$$

and the overseas account in balance because the deficit on current account (b) is financed through capital inflows.

B is nevertheless still only a quasiequilibrium because the monetary authorities are creating domestic money at a rate $(e_c - e_f)b$ - which is negative. So, as before, the money supply falls moving LM to LM' and the economy to a sustainable equilibrium at c.

B. Monetary Policy

(i) non-intervention



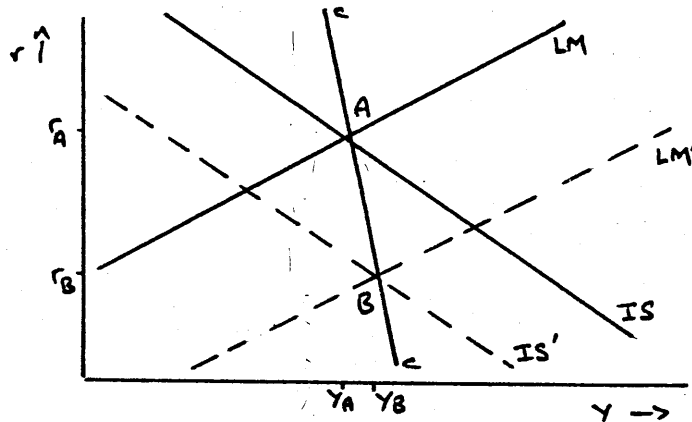
Our economy is initially at equilibrium at A as above. The money supply is now expanded to LM'. The Economy moves to B. The Financial rate now depreciates until $e_f/e_c r_B = r^*$ at which point B is a quasiequilibrium. But the current account is in deficit and in long term the outflow of reserves causes the money supply to fall and the economy moves back to A.

(ii) neutral intervention

Using the same diagram, initial equilibrium etc as above, B is again a quasiequilibrium where the balance of payments deficit (b) is now financed through the financial market. But at B the rate of creation of money $(e_c - e_f)b$ is positive - so the money supply continues indefinitely to expand and there is no long term equilibrium.

C. Anti-Swoboda

Decaluwe and Steinherr's answer to Swoboda's claim that, under non-intervention, it is not possible to use fiscal and monetary policy in a dual market to affect the equilibrium value of Y .

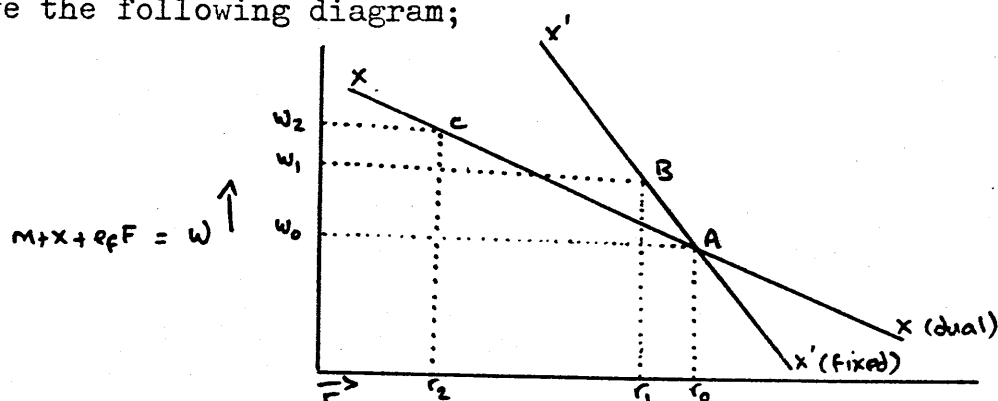


Taking interest payments on overseas debt into account the current account (balanced along line cc) is interest rate dependent. Now moving from LM to LM' and IS to IS' we move our economy to point B . e_f again adjusts until $e_f/e_c r_B = r^*$ and the current account is balanced since B lies on cc . So B is a sustainable equilibrium with $Y_B \neq Y_A$

Appendix II

Aizenmann simplified:

We have a small open economy with fixed output, wealth effect etc. Domestic wealth W consists of three assets; domestic bonds (of which there is a fixed quantity X), domestic money M and overseas bonds to total (domestic currency) value $e_f F$, where e_f the financial exchange rate may be fixed and unified with the commercial rate, or floated apart from it. Holders preferences amongst these three types of asset will be affected in obvious ways by variations in r , the domestic interest rate. In particular a fall in r will lead to disposals of domestic bonds, some greater disposition to hold money, and purchases of overseas bonds which - where e_f floats - will simply result in a bidding down of the financial exchange rate. We then have the following diagram;

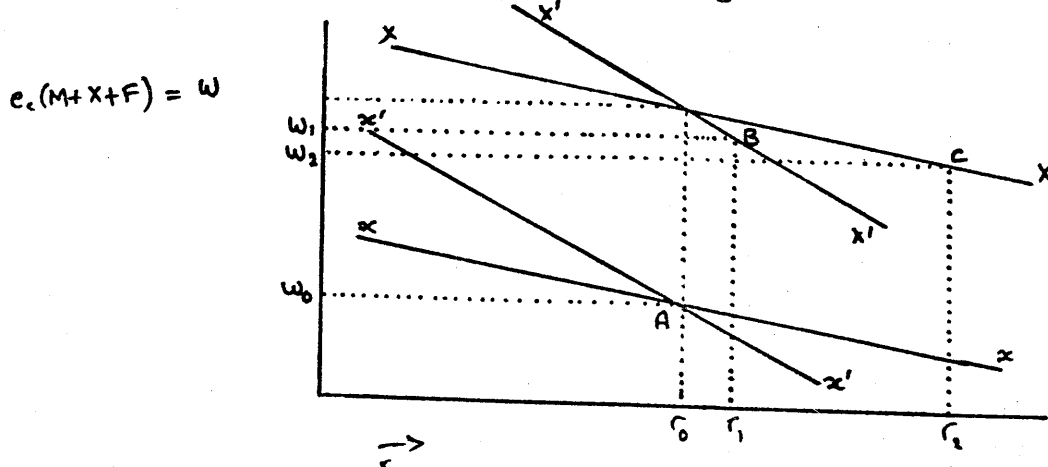


Here the curve XX gives those values of r which, at varying levels of W and under a system of dual exchange rates, results in holdings of domestic bonds of just X . As noted above this curve will be downward sloping. $X'X'$ is the corresponding locus under fixed rates. The key point to note is that $X'X'$ is steeper than XX . This is because under dual rates the rate of return on domestically held overseas bonds ($= e_c r^* / e_f$) will diminish as r diminishes (and so, as noted above, e_f rises)

whereas under fixed rates it is fixed. Thus the disposition to continue to hold domestic bonds as interest rates fall will be greater under dual rates than under fixed rates. The point A is of course where $e_c = e_f$, the long term equilibrium.

Now suppose there is a rise in domestic money supply δM . Under fixed rates this will raise wealth from w_0 to $w_1 = w_0 + \delta M$. Under dual rates it will raise wealth further, say to w_2 , because the resulting fall in interest rates will lead to a rise in e_f and so in e_f^F . So under fixed rates the economy moves initially to point B, and under dual rates to point C i.e. under dual rates the rise in interest rates is sharper. The wealth effect will now result in a current account deficit which will gradually restore the economy to equilibrium - again it is clear that this will operate more sharply in the case of dual rates than fixed rates (assisted by the fact that under fixed rates there will also have been some conversion of domestic money into overseas bonds resulting in an extra inflow of interest payments).

A very similar argument, with the same result, can be constructed for a devaluation (i.e. a rise in e_c) coupled with a rise in the same proportion in X (to neutralise the effect on the long term equilibrium). Indeed, if the shift from X to $e_c X$ (with $e_c > 1$) results in the curve xx moving to XX , and $x'x'$ moving to $X'X'$ we have (assuming constant returns to scale)



under fixed rates the initial breakdown of wealth becomes

$W_1 = M + e_c X + e_c F$ and the economy moves to a point

like B. Under dual rates $W_2 = M + e_c X + e_f F$

and $e_f < e_c$ because if $e_f \geq e_c$ the (domestic) value of overseas bonds held is greater as a proportion of total domestic wealth than at position A, while the return on them is the same or less (and the return on domestic bonds is certainly higher). So the economy moves to a point like C.

CHAPTER TWO: A RATIONAL EXPECTATIONS MODEL OF A DUAL EXCHANGE MARKET

I. Introduction

The model below goes beyond previous rational expectations models of dual exchange markets (notably those described in Chapter I, of Flood and Marion (1982) and Cumby (1984) in a number of ways;

- a) it is the first rational expectations model of an economy with market power which allows for a precise comparison of the insulation properties of dual exchange rates with other exchange rate regimes;
- b) it explicitly incorporates the possibility of Government intervention between the two exchange markets;
- c) it allows for an analysis of the effectiveness of dual rates against other exchange systems in circumstances where only imperfect information is available.

The chief conclusions that follow from the model are that for an open economy producing a differentiated product and with sensible parameter values a dual exchange rate system is more effective than either fixed or floating unified rates in insulating domestic economic activity from the effect of external capital movements, and no less effective in insulating it from external price changes. We derive this conclusion both when overseas prices and interest rates oscillate randomly about some fixed and known equilibrium (or trend) and when they follow a random walk on which policymakers only have imperfect information. It also remains true, but diminishingly so, if the Government intervenes between the two markets e.g. by transferring a proportion of the balance of payments surplus on current transactions from the commercial market foreign exchange pool to the financial market pool.

This chapter is arranged as follows. Part II sets up a general model of an open economy which is adaptable for the three exchange rate régimes under examination. Part III describes those régimes. In parts IV, V and VI the model is solved under those régimes on the assumption that external prices and interest rates are set by white noise processes about some fixed point. Part VII compares those solutions and part VIII extends the results to the case where external prices and interest rates follow a random walk with imperfect information.

II The Model

Our model is an adaptation and elaboration of the model of Dornbusch and Fischer (1980), in which the exchange rate is set by capital movements, and balance on the current account is achieved, over time, by the effect of asset movements working through the wealth effect.

Specifically we assume an open economy producing a differentiated product. We allow domestic holdings of two types of asset: foreign bonds and foreign money. Domestic money supply is assumed to consist of the latter plus a constant element of domestic credit. Domestically held domestic bonds are not excluded but are assumed to make a net contribution of zero to domestically held wealth (since their effect on the behaviour of borrowers and lenders will be equal and opposite). The same applies to the domestic capital stock and domestic credit and the Government sector (where holdings of Government bonds are offset by expected future tax liabilities). Following Dornbusch and Fishcher we assume that Foreign Bonds are denominated in real terms, but unlike them we assume that they return interest

at the current overseas rate (which seems likely to be a more realistic approximation than their assumption that they return interest at a fixed rate).

Now, allowing capital and current foreign exchange transactions to take place at different rates we have the following equations;

$$1) \quad y_t = \mu (P_t - E_{t-1}(P_t))$$

$$2) \quad r_t = r_t^* - (1+r_0)e_t^f + E_{t-1}(e_{t+1}^f) + r_0 E_{t-1}(e_{t+1}^f)$$

where $e_t^f = e_t^f$ or e_t^c , see below

$$3) \quad m_{t+1} - m_t + \beta_1(b_{t+1} - b_t) = \beta_1 r_t^* + \beta_1 r_0 b_t - s_2 m_t - s_2 \beta_1 (e_t^f - e_t^c + b_t) - \beta_1 (s_2 - r_0) P_t^* + \beta_2 (P_t - e_t^c + y_t) + \beta_3 r_t$$

$$4) \quad m_{t+1} - m_t + \beta_1(b_{t+1} - b_t) = \beta_1 r_t^* + \beta_1 r_0 b_t + \gamma_1 (e_t^c + P_t^* - P_t) - \gamma_2 y_t$$

$$5) \quad q(e_t^c + m_t) - P_t = -\alpha r_t + y_t$$

Where, at this stage, we assume all variables are small divergences from equilibrium, and all are logs except r_t, r_t^*, r_0 . Notation is as follows;

y_t = output

P_t = domestic price level

r_t = domestic interest rate

r_t^* = overseas interest rate (assumed exogenous)

e_t^f = exchange rate for capital transactions ('financial' exchange rate)

e_t^c = exchange rate for current transactions ('commercial' exchange rate)

r_0 = equilibrium domestic interest rate

- m_t = domestic holdings of foreign currency
 b_t = domestic holdings of overseas bonds
 P_t^* = overseas price level (assumed exogenous)

The explanation (as opposed to the derivation, which being unaesthetic is relegated to appendix I) of these equations is as follows;

Equation 1 the standard Lucas-Sargent surprise supply function

Equation 2 is the interest rate parity equation. It assumes perfect capital mobility and equates the expected returns on capital held domestically (on the lefthand side) with that for capital held overseas (on the right). The value of e'_t will depend on whether interest on domestically held overseas bonds is paid through the financial market (when $e'_t = e_t^f$) or through the commercial market (when $e'_t = e_t^c$). We have called the latter possibility an 'Italian type' dual market, and the former a 'Belgian type'. We will henceforth confine our attention to Belgian type dual markets. The major differences for Italian type markets are noted in appendix III.

Equation 3 equates, at overseas prices, accumulation of overseas assets (on the lefthand side) to net domestic deferred consumption (on the right). The latter comprises the following components;

- i) interest on domestic holdings of overseas bonds

$$\beta_1 r_t^* + \beta_1 r_0 (b_t + P_t^*)$$
- ii) the fall in consumption (so rise in deferred consumption) brought about, through the wealth effect, by a change in holdings of foreign assets;

$$-s_2 m_t - s_2 \beta_1 (e_t^f - e_t^c + P_t^* + b_t)$$

iii) the rise in domestic saving:

$$\beta_2 (P_t - e_t^c + y_t)$$

iv) the fall in domestic investment: $\beta_3 r_t$

Equation 4 is the demand equation (in terms of overseas prices) for traded goods. It says that the net accumulation of overseas assets (the lefhand side) is equal to the sum of

i) interest on domestic holdings of overseas bonds;
 $\beta_1 r_t^* + \beta_1 r_0 b_t$ (see Appendix for explanation as to why this differs from corresponding term in equation 3)

ii) a term proportional to the improvement in the terms of trade: $\gamma_1 (e_t^c + P_t^* - P_t)$

(Note that by assuming $\gamma_1 > 0$ we are assuming the Marshall-Lerner condition).

iii) a term negatively proportioned to domestic income:
 $-\gamma_2 y_t$

Equation 5 is the demand for money equation where $\eta < 1$ reflects the proportionality between the equilibrium level of domestic credit and holdings of foreign money.

We also have the following comments on the parameters;
a) β_1 is the ratio of the value of foreign bonds to the value of foreign money held in equilibrium, r_0 is the equilibrium domestic interest rate and s_2 the proportion of wealth transformed into current consumption. So $s_2 < 1$. Quite clearly if $r_0 > s_2$ then wealth earns interest faster than it is spent and our model is unstable. We exclude this possibility and in fact assume r_0 reasonably small, small enough to exclude

various other, similar, possibilities for instability which will appear as our analysis proceeds.

b) $\beta_2 = s_2 + (s_2 - r_0) \beta_1$ (see appendix)

c) since all our variables except r, r^* are logs all of our parameters can be expected to be $O(1)$ except for α and β_2 which can be expected to be $O(r_0^{-1})$ i.e. in general those two parameters will be significantly larger than the others.

d) $\gamma_1 < 1$ (see appendix)

III The Exchange Rate Régimes

Our model is incomplete. We have five equations in seven unknowns $(r_t, p_t, m_t, v_t, y_t, e_t^i, e_t^f)$.

The Government is free to impose two additional constraints by its choice of exchange rate régime. We consider three possibilities.

a) fixed unified rates;

The Government imposes $e_t^i = e_t^f = 0$. Equation 2

becomes $r_t = r_t^*$

b) floating unified rates

The Government imposes $e_t^i = e_t^f$ but allows the value of this variable to be set in each period by the market. This still leaves the Government one degree of freedom which we assume, plausibly, that it uses to control the monetary impact of external events by neutralising a constant proportion of any variation in domestic holdings of overseas assets. Specifically we assume that, through Government action, a fixed proportion of any variation from equilibrium of the value

/of

of the national portfolio of overseas assets will be held as bonds (which of course count as part of wealth). So putting

$$C_t = m_t + \beta_t b_t$$

$$\beta_t b_t = \theta C_t \quad \text{and} \quad m_t = (1-\theta) C_t$$

c) dual rates

In conformity with the way this kind of system has normally been run (which is intended to give price stability to traders while not attempting to establish a fixed financial exchange rate in the face of the powerful force of external capital movements) the Government fixes the commercial rate $e_t^c = 0$ but allows e_t^f to float. Again this leaves the Government free to impose one other constraint. Again we assume the Government chooses to impose a fixed intervention rate (as introduced in Lanyi (1975)) which is the proportion of any surplus (or deficit) on the commercial market which the Government uses for purchases (or sales) of foreign bonds which are then added to (or subtracted from) the pool available to financial market. So again

$$\beta_t b_t = \theta C_t$$

$$\text{and} \quad m_t = (1-\theta) C_t$$

Thus $\theta = 0$ in Lanyi's terminology, is a policy of non-intervention and $\theta = 1$ is a policy of neutral intervention under which all of any imbalance on the commercial market is transferred to the financial market. It is worth recalling again Lanyi's observation that the overall foreign account is only in balance under a policy of neutral intervention and his conclusion that such a policy therefore offers the only sustainable way of running a dual market régime. We will return to this below.

IV Solution of the model under fixed rates

We assume here, and for the next three sections, that p_t^* and r_t^* are white noise processes about $p^* = 0$ and $r^* = 0$. Thus we need only solve our model for the initial impact on, and subsequent time path of, the economy following a random single period fluctuation from zero in p_0^* and r_0^* - the complete solution then being the sum of those individual solutions over all past time.

So we have $p_t^* = r_t^* = 0 \quad \forall t \neq 0$

so for $t \geq 1 \quad y_t = r_t = 0 \quad (\text{from equations (1) and (2)})$

so for $t \geq 1 \quad q^m t = p_t \quad (\text{from (5)})$

where, equating (3) and (4), for $t \geq 1 \quad u_t = \frac{\beta_2 + \delta_1 - q^{-1} s_2}{s_2 \beta_1} p_t$

substituting back into (4) we get

$$\text{for } t \geq 1 \quad p_{t+1} = \left[1 - \frac{\gamma_1 s_2 - (\beta_2 + \delta_1 - q^{-1} s_2) r_0}{\beta_2 + \delta_1} \right] p_t$$

So, recalling $s_2 < 1$ our model is stable provided that r_0 is small enough which (as promised above) we assume.

We can also solve the model for period zero to find at $t = 0$;

$$p = \frac{\gamma_1 + \beta_1 (s_2 - r_0)}{\gamma_1 + \mu \gamma_2 + \beta_2 (1 + \mu)} p^* - \frac{\beta_2}{\gamma_1 + \mu \gamma_2 + \beta_2 (1 + \mu)} r^*$$

and of course $r = r^*$

Since exchange rates are fixed it is rather easy to see what is going on here. An unexpected one period, e.g. rise, in overseas prices leads (in period zero) to an incipient rise in demand for domestic production (both through increased

exports and a fall in imports). This leads to a rise in domestic prices which is nevertheless not enough entirely to offset the improvement in the terms of trade. In order to meet the increased domestic money demand there is also a shift in the portfolio of overseas assets held from bonds to money, and there is some accumulation of overseas assets due to the improved terms of trade. Then in subsequent periods the terms of trade have turned negative (since $p^* = 0$ again) and there is extra domestic spending generated by the wealth effect, so the extra assets accumulated gradually run down as the domestic prices gradually slide back to their equilibrium level.

A similar process is initiated by an unexpected rise in r^* (which through infinite capital mobility leads to an instant and equal rise in r) except that now it brings about a fall in domestic consumption (via a fall in investment) and a fall in domestic money demand - and hence a fall in domestic prices. The resulting inflow of overseas assets and greater demand on domestic production as a result of the positive terms of trade cause domestic prices to rise to above equilibrium and then to slide gradually back to equilibrium as described above.

Finally for this section we note that if we add an exogenous variation in domestic money supply, dt to the left hand side of equation 5, and solve for the initial impact of this we rediscover the result of Mundell, $p_0 = r_0 = 0$ and $m_0 = -q^1 dt$ i.e. there is an offsetting outflow of holdings of overseas money.

V Solution of the model under unified floating rates

We put $e_t = e_t^e = e_t^f$ and recall that we have defined

$$c_t = m_t + \beta_1 b_t \quad \text{and imposed} \quad m_t = (1-\theta) c_t$$

$$\text{and} \quad \beta_1 b_t = \theta c_t.$$

Now if there is a one period shock to p^* and r^* at $t=0$ then for $t \geq 1$ the economy will follow its expected course, and in particular $y_t = 0$ So for $t \geq 1$

$$q(1-\theta) c_t - p_t = -q e_t - \alpha r_t \quad (\text{from (5)})$$

$$\text{and} \quad s_2 c_t - (\beta_2 + \gamma_1) p_t = -(\beta_2 + \gamma_1) e_t + \beta_3 r_t \quad (\text{from (3) and (4)})$$

combining these we find;

$$[q(1-\theta)(\beta_2 + \gamma_1) - s_2] c_t = (1-q)(\beta_2 + \gamma_1) e_t - [\alpha(\beta_2 + \gamma_1) + \beta_3] r_t$$

$$\text{and} \quad [q(1-\theta)(\beta_2 + \gamma_1) - s_2] p_t = q[(1-\theta)(\beta_2 + \gamma_1) - s_2] e_t - [s_2 \alpha + (1-\theta)q\beta_3] r_t$$

We now use these formulae to substitute for c_t and p_t back into equation (4). In order to tidy things up put

$$\alpha(\beta_2 + \gamma_1) + \beta_3 = a > 0$$

Then we find

$$\begin{aligned} \alpha r_{t+1} - [(1+r_0\theta)a - \gamma_1(s_2\alpha + (1-\theta)q\beta_3)] r_t \\ - (1-q)(\beta_2 + \gamma_1) e_{t+1} + (1-q)[(1+r_0\theta)(\beta_2 + \gamma_1) - s_2\gamma_1] e_t = 0 \end{aligned}$$

now we have from equation(2) for $t \geq 1$

$$r_t = (1+r_0)(e_{t+1} - e_t)$$

and we recall that a quadratic equation $ax^2 + bx + c$ with $a > 0$, $b < 0$, $c > 0$ has just one root with modulus less than one if and only if $a + b + c < 0$. Now substituting for r_t in the above equation to get

$$Ae_{t+2} + Be_{t+1} + Ce_t = 0$$

it is easy to check that $A > 0$, $B < 0$, $C > 0$

and

$$\begin{aligned} A+B+C &= (1-q)[(1+r_0\theta)(\beta_2 + \gamma_1) - s_2\gamma_1 - (\beta_2 + \gamma_1)] \\ &= (1-q)[r_0\theta(\beta_2 + \gamma_1) - s_2\gamma_1] \end{aligned}$$

Hence on the continuing assumption that r_0 is small enough not to cause us trouble (in this case $r_0 < \frac{s_2 \gamma_1}{\beta_2 + \gamma_1}$) we find that our system has one stable and one unstable root, so that by the standard theory of rational expectations models it finds and follows its unique stable path.

Armed with this assurance, and denoting the stable eigenvalue by λ_1 , we can now explicitly derive the solution of our model as follows;

a) solve for period 0. Since at this period all expectations are zero, and $c_0 = 0$, this boils down to an exercise in solving linear equations.

Putting

$$K = [(1+r_0)\alpha - \gamma] [(1+\mu)(\beta_2 + \gamma_1) + \mu(\gamma_2 - \gamma_1)] + (1+\mu)[\beta_2 + \gamma_1 + (1+r_0)\beta_3]$$

and noting that since $\alpha = O(r_0^{-1})$, $\alpha > \frac{\gamma}{1+r_0}$ so $K > 0$

thus we have at period 0;

$$p = \frac{[(1+r_0)\alpha - \gamma][\gamma_1 + \beta_2 - s_2]}{K} p^* + \frac{\gamma\beta_3 + \alpha(\beta_2 + \gamma_1)}{K} r^* \quad (\text{and } y = \mu p)$$

$$e = -\frac{(1+\mu)(\gamma_1 + \beta_2 - s_2)}{K} p^* + \frac{(1+\mu)(\gamma_1 + \beta_2 + \beta_3) + \mu(\gamma_2 - \gamma_1)}{K} r^*$$

$$r = \frac{(1+r_0)(1+\mu)(\gamma_1 + \beta_2 - s_2)}{K} p^* + \frac{[(1+r_0)(\alpha - 1) - \gamma][(1+\mu)(\beta_2 + \gamma_1) + \mu(\gamma_2 - \gamma_1)] - (1+\mu)(\beta_2 + \gamma_1)}{K} r^*$$

It is worth commenting that all of the coefficients in these expressions are unambiguous in sign (the only questionable case, the r^* coefficient in the expression for r , being handled by our assumption that α is reasonably large - in particular $\alpha > (1+r_0)^{-1}(2 + \gamma + \mu)$ so the coefficient is always positive).

From these it is possible to derive expressions for the terms of trade and accumulation of overseas assets in the first period (from equation 4);

$$p^* + e - p = \frac{[(1+r_0)\alpha - \gamma][\mu(\beta_2 + \gamma_1) + s_2] + (1+\mu)[s_2 + (1+r_0)\beta_3]}{K} p^*$$

$$+ \frac{(1+\mu - \gamma)\beta_3 + \mu(\gamma_2 - \gamma_1) - (\alpha - 1 - \mu)(\gamma_1 + \beta_2)}{K} r^*$$

$$c_1 = \frac{[(1+r_0)\alpha - \gamma] [\mu\beta_2(\gamma_1 - \delta_2) + s_2(\gamma_1 + \mu\gamma_2)] + (1+\mu)\gamma_1 [s_2 + (1+r_0)\beta_2]}{K} p^d$$

$$+ \frac{(1+\mu)[(\alpha-1)[(1+r_0)\beta_1 - \gamma_1] - (1+\gamma)\beta_1(\delta_1 + \beta_2) + \mu[(\alpha-1)((1+r_0)\beta_1 - \gamma_1) - \gamma\beta_1 - \alpha\beta_2 - \gamma\beta_3](\delta_1 - \gamma_1) + (1+\mu)(1-\gamma)\gamma_1\beta_3}{K} r^d$$

From these expressions it is clear that (eg) a rise in overseas prices brings about an unambiguous initial improvement in the terms of trade and an inflow of overseas assets except in the very special case where the expansion of domestic income sucks in more than enough imports to offset the improved terms of trade (ie γ_2 relatively large). The initial effect of a rise in r^* is more equivocal. Through capital mobility this will place upward pressure on domestic interest rates and hence, in order to balance the demand for money, on domestic prices. So the terms of trade seem set to deteriorate. But the rise in domestic interest rates also causes a fall in domestic demand through a fall in investment. This can only be offset by increased exports via a more competitive exchange rate. And this pulls the terms of trade the other way. The net effect depends on the precise value of the parameters in our model. The terms of trade will tend to improve if the fall in investment effect dominates (β_2 relatively large) and to deteriorate if the monetary balance effect dominates (α relatively large). The question of whether there is a rise or a fall in overseas assets will similarly depend on precise parameter values.

b) for periods $t > 0$ by assumption \hat{p}^* and r^* revert to zero and the general theory of rational expectations models assures us that expectations will be such that the economy will place itself on a path which converges, with speed λ_1 back to its original equilibrium. In particular we have already calculated c_1 the accumulation/decumulation of overseas assets resulting from the original disturbance. It must now be the case for $t > 1$ that

$$c_t = c_1 \lambda_1^{t-1}$$

but for equations (3) and (4)

$$(\delta_1 + \beta_2) c_{t+1} - [(\delta_1 + \beta_2)(1 + r_0 \theta) - \delta_1 s_2] c_t = \gamma_1 \beta_3 r_t$$

so

$$r_t = - \frac{(\delta_1 + \beta_2)(1 + r_0 \theta - \lambda_1) - \delta_1 s_2}{\gamma_1 \beta_3} c_1 \lambda_1^{t-1}$$

and from equation 2, recalling that on the surprise free path all expectations are met

$$r = (1 + r_0) [e_{t+1} - e_t]$$

and, as for c_t , we must have $e_{t+1} = \lambda_1 e_t$

so

$$e_t = \frac{-r_t}{(1 + r_0)(1 - \lambda_1)} = \frac{(\delta_1 + \beta_2)(1 + r_0 \theta - \lambda_1) - \delta_1 s_2}{(1 + r_0)(1 - \lambda_1) \gamma_1 \beta_3} c_1 \lambda_1^{t-1}$$

and from equation 4

$$p_t = e + \gamma_1' [(1 + r_0 \theta) c_t - c_{t+1}]$$

so

$$p_t = \frac{(1 + r_0 \theta - \lambda_1) [\gamma_1 + \beta_2 + \beta_3 (1 + r_0)(1 - \lambda_1)] - \delta_1 s_2}{(1 + r_0)(1 - \lambda_1) \gamma_1 \beta_3} c_1 \lambda_1^{t-1}$$

the signs of these various expressions will depend on the parameters. This is not so for the terms of trade which, again from equation 4, we can determine to be;

$$(e - p)_t = - \frac{1 + r_0 \theta - \lambda_1}{\gamma_1} c_1 \lambda_1^{t-1}$$

i.e. if $c_1 > 0$ (so the initial disturbance has lead to an accumulation of assets) then the terms of trade are unambiguously negative - as we would expect since the path back to equilibrium

entails a running down of holdings of overseas assets
i.e. a trade deficit.

We can get a slightly clearer insight into the dynamics of this system, and in particular the role played by the intervention rate by looking at our results diagrammatically.

Along the perfect foresight return to equilibrium we can eliminate p and y from the equations of our model to find;

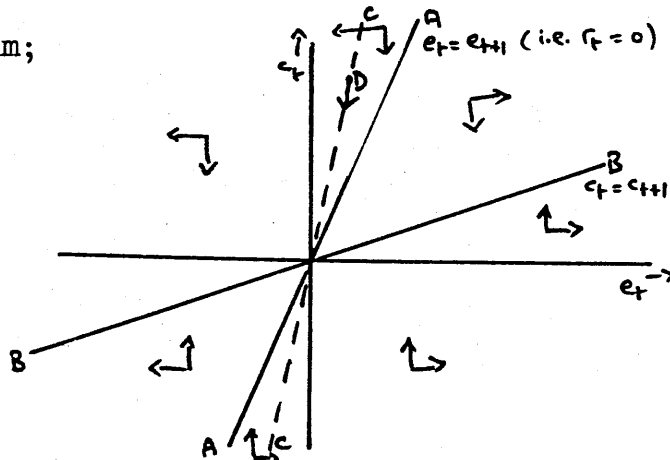
$$A) [q(1-\theta)(\delta_1+\beta_2)-s_2]c_t - (\delta_1+\beta_2)(1-q)e_t = -[\alpha(\delta_1+\beta_2)+\beta_3](1+r_0)(e_{t+1}-e_t)$$

$$B) [q(1-\theta)\gamma_1\beta_3+\alpha\gamma_1s_2-r_0\theta(\alpha(\delta_1+\beta_2)+\beta_3)]c_t - \gamma_1\beta_3(1-q)e_t = -[\alpha(\delta_1+\beta_2)+\beta_3][c_{t+1}-c_t]$$

Equation A gives the locus along which $e_t = e_{t+1}$ and $r_t = 0$.

Equation B gives the locus along which $c_t = c_{t+1}$.

When $\theta \leq 1 - \frac{s_2}{q(\delta_1+\beta_2)}$ the $r_t = 0$ locus has positive slope and we have the following phase diagram;

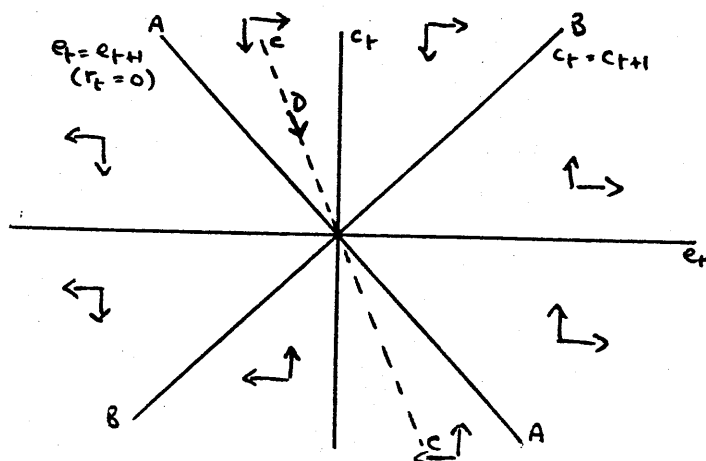


Above and to the left of AA the level of holdings of foreign assets is relatively high, and is largely held as money. Hence domestic interest rates are low and (to equalise domestic and international returns on capital) the exchange rate is appreciating. So $e_t > e_{t+1}$ and our economy moves leftwards. Similarly below and to the right of AA $e_{t+1} > e_t$ and our economy moves rightward. Turning our attention to BB, above and to the left of it the level of holdings of foreign assets is high. This raises domestic spending and so imports so $c_{t+1} < c_t$ and our economy moves down. Below and to the

right of BB $c_{t+1} > c_t$ and it moves up. It is easy to check that (provided r_0 is small enough) AA is steeper than BB and that as θ increases both become steeper. The perfect foresight stable trajectory must be a line something like CC along which accumulated overseas assets and the exchange rate both gradually slide back to their equilibrium state. It is worth commenting that for the values of θ to which this diagram corresponds starting from a point like D the exchange rate gradually appreciates as holdings of overseas assets diminish - although throughout the process, as noted above, the terms of trade remain negative.

For higher levels of intervention, i.e. when $\theta > 1 - \frac{s_2}{q(\gamma_1 + \beta_2)}$ (or of course for all θ if $s_2 > q(\gamma_1 + \beta_2)$)

the slope of the $r_t = 0$ locus becomes negative and our diagram becomes;



our stable trajectory is now C-C so that starting from a point like D the exchange rate depreciates as holdings of overseas assets diminish. The reason for this change is that at high levels of intervention the accumulation of overseas assets is held largely as bonds; its impact on current spending through the wealth effect (tending to raise domestic interest rates by diminishing the funds available for investment) therefore outweighs its contribution to the money supply (which tends to cut domestic interest rates). But if domestic interest rates are above equilibrium (and so falling) perfect capital mobility will

equalise domestic and overseas returns by bidding up the exchange rate, and allowing it to depreciate. (If $s_1 > q(\gamma_1 + \beta_2)$ then the wealth effect is so powerful that it outweighs the monetary expansion even with a policy of non-intervention.)

Finally for this section we note two other points;

a) effect of the level of intervention on the rate of return to equilibrium.

If we denote the unstable root of our system quadratic (p.46) by λ_2 then inspection of the equations coefficients shows, recalling our assumption that θ is small enough not to get in the way (in particular $\theta < \frac{\gamma_1 \beta_3}{\alpha + 1 - \gamma}$), that as θ increases $\lambda_1 + \lambda_2$ increases and $(1 - \lambda_1)(1 - \lambda_2)$ (which is $\frac{A + \beta + c}{A}$) increases. So, recalling that $\lambda_2 > 1 > \lambda_1 > 0$, as θ increases $(1 - \lambda_1)(\lambda_2 - 1)$ decreases, so either λ_1 increases or both λ_1 and λ_2 decrease. But the latter possibility is contradicted by the fact that $\lambda_1 + \lambda_2$ is increasing. So as θ increases λ_1 increases, i.e. as the rate of intervention increases the rate of return to equilibrium gets slower. This result sits well with common sense since one of the forces tending to restore equilibrium after e.g. an accumulation of overseas assets is the monetary effect of these assets pushing up domestic prices and so accelerating the outflow. If a high proportion of the accumulation of assets are demonetised (θ high) then this effect is weakened.

b) The effect of domestic monetary disturbance.

We may include in our model exogenous white noise variations in domestic credit. Equation 5 then becomes

$$q(e_t^c + m_t) + d_t - p_t = \alpha r_t + y_t$$

where d_t assumed small, is the log exogenous shift in domestic credit at period t . We can solve for the impact of this as above, assuming a single period disturbance at period 0. We find at period 0;

$$p = \frac{\beta_2 + \gamma_1 + (1+r_0)\beta_3}{\kappa} d_0 \quad \text{positive}$$

$$e = \frac{\beta_2(1+\mu) + \gamma_1 + \mu\gamma_2}{\kappa} d_0 \quad \text{positive}$$

$$r = - \frac{(1+r_0) [\beta_2(1+\mu) + \gamma_1 + \mu\gamma_2]}{\kappa} d_0 \quad \text{negative}$$

terms of trade:

$$e-p = \frac{\mu(\beta_2 + \gamma_2) - (1+r_0)\beta_3}{\kappa} d_0 \quad \text{almost certainly negative}$$

$$c_1 = \frac{\mu\beta_2(\gamma_1 - \gamma_2) - (1+r_0)(1+\mu\gamma_2)\beta_3}{\kappa} d_0 \quad \text{almost certainly negative}$$

we will refer back to these results below.

VI Solution of the model under dual rates

In this case we put $e_t^c = 0$ and (to save on superscripts) write $e_t = e_t^f$. As before we also impose

$$m_t = (1-\theta)c_t$$

and $\beta_1 b_t = \theta c_t$

Again, following a one period shock at period zero in p^* and r^* then for $t \geq 1$ the economy will follow its expected course, so in particular $y_t = 0$. Hence for $t \geq 1$ from equation (5)

$$q(1-\theta)c_t - p_t = -\alpha r_t$$

so that combining (3) and (4) to eliminate c_{t+1} and substituting for p_t from the above;

$$[(\beta_2 + \gamma_1)q(1-\theta) - s_2]c_t = -[\alpha(\beta_2 + \gamma_1) + \beta_3]r_t + s_2\beta_1 e_t$$

whereas substituting for p_t from equation (5) in equation (4)

$$c_{t+1} - [1 + r_0\theta - \gamma_1 q(1-\theta)]c_t = -\alpha\gamma_1 r_t$$

we now substitute into this, from the preceding equation,

getting rid of c_t, c_{t+1} to get;

$$\alpha r_{t+1} - s_2\beta_1 e_{t+1} - E\alpha r_t + E s_2\beta_1 e_t = \alpha\gamma_1 D r_t$$

where, as before.

$$a = \alpha(\beta_2 + \gamma_1) + \beta_3 > 0$$

$$\text{and } E = 1 + r_0 \theta - q\gamma_1(1-\theta) > 0$$

$$D = (\beta_2 + \gamma_1)q(1-\theta) - s_2$$

so substituting from (2)

$$r_t = (1+r_0)[e_{t+1} - e_t]$$

we find

$$ae_{t+2} - \left[a + \frac{s_2\beta_1}{1+r_0} + aE + \alpha\gamma_1 D \right] e_{t+1} + \left[aE + \alpha\gamma_1 D + \frac{Es_2\beta_1}{1+r_0} \right] e_t = 0$$

which is the stability equation for our system. Clearly the coefficient in e_{t+2} is positive. That in e_{t+1} is negative because $a > \alpha\beta_2 > \alpha\gamma_1 s_2$ (since $\gamma_1 < 1$) - and $\alpha\gamma_1 s_2$ is the negative component of $\alpha\gamma_1 D$. It is slightly more complicated to demonstrate, but nevertheless true, that the coefficient in e_t is positive. The first point to note is that the coefficient of θ in the expression when expanded is positive - so the expression is minimal when $\theta = 0$. But then it becomes

$$\alpha(\beta_2 + \gamma_1) + (1 - q\gamma_1)\beta_3 - \alpha\gamma_1 s_2 + \frac{Es_2\beta_1}{1+r_0}$$

which is positive for the same reason as above.

Thus we can apply the same test for the existence of a unique stable root as we did for unified floating rates. The sum of the coefficients in the stability equation is

$$(E-1) \frac{s_2\beta_1}{1+r_0}$$

so for our system to have a stable rational expectations solution we require

$$E < 1$$

$$\text{i.e. } \theta < \frac{q\gamma_1}{r_0 + q\gamma_1}$$

Which, while it does not look like a particularly restrictive condition in the level of intervention θ nevertheless gives us the important result that in a Belgian system of dual exchange rates too high a level of intervention, and in particular a policy of neutral intervention is destabilising. This result (while in flat disagreement with that of Decaluwe and Steinherr (1976b), referred to in Chapter I) is in accord with economic intuition. A major equilibrating mechanism in this model, in response to e.g. a rise in overseas assets is the rise in money supply leading to a rise in domestic prices, a deterioration in the terms of trade and so a faster outflow of these assets. A high level of intervention damps this mechanism by breaking the link between accumulation of overseas assets and a rise in the money supply.

We can now solve our model explicitly in the same two stages as we used for unified rates;

a) solve for period 0. Again all expectations are zero and $c_0 = 0$. So putting

$$L = \alpha(1+r_0) [(1+\mu)(\beta_2 + \gamma_1) + \mu(\gamma_2 - \gamma_1)] + (1+\mu) [s_2\beta_1 + (1+r_0)\beta_3] > 0$$

we find at period 0;

$$p = \frac{(1+r_0)\alpha [\delta_1 + \beta_2 - s_2]}{L} p^* + \frac{\alpha s_2 \beta_1}{L} r^*$$

$$r = \frac{(1+\mu)(1+r_0) [\gamma_1 + \beta_2 - s_2]}{L} p^* + \frac{(1+\mu)s_2\beta_1}{L} r^*$$

$$\theta = - \frac{(1+\mu) [\gamma_1 + \beta_2 - s_2]}{L} p^* + \frac{\alpha [(1+\mu)(\beta_2 + \gamma_1) + \mu(\gamma_2 - \gamma_1)] + (1+\mu)\beta_3}{L} r^*$$

from which we can calculate the initial impact on the terms of trade (which do not of course involve the financial exchange rate)

$$p^* - p = \frac{\alpha(1+r_0) [\mu(\beta_2 + \gamma_2) + s_2]}{L} p^* + \frac{(1+\mu) [s_2\beta_1 + (1+r_0)\beta_3]}{L} p^* - \frac{\alpha s_2 \beta_1}{L} r^*$$

and the initial accumulation of overseas assets;

$$c_1 = \frac{\alpha(1+r_0) [\mu(\delta_1 - \delta_2)\beta_2 + \gamma_1 s_2 + \mu \delta_2 s_2]}{L} + \frac{(1+\mu)\gamma_1 [s_2\beta_1 + (1+r_0)\beta_3]}{L} p^* - \frac{(\gamma_1 + \mu \gamma_2) \alpha s_2 \beta_1}{L} r^*$$

These expressions show that the impact of an unexpected (e.g.) rise in p^* in a dual rate régime is very similar to its impact under a fixed exchange rate. The increased demand for domestic output leads to a rise in domestic prices which is nevertheless not enough entirely to offset the improvement in the terms of trade. The trade exchange rate by assumption is fixed. So there is an accumulation of overseas assets (unless Y_1 is so large that the imports sucked in by increased income outweigh the effect of the improved terms of trade). There is, however, one mechanism which operates in this case but not in the case of fixed rates. The increased demand for domestic output also leads to a rise in domestic interest rates which (by diminishing domestic investment) means that domestic prices rise less than in the fixed rates case. The differential which thus emerges between domestic and overseas interest rates is accommodated by an upvaluation of the financial exchange rate (in the expectation that it will then decline back to equilibrium).

A rise in overseas interest rates has a very different effect from what happens in the case of unified rates. The incipient differential between domestic and overseas interest rates leads to a bidding up in the price of overseas bonds, and so a rise in the financial exchange rate. Through the wealth effect there is then a rise in domestic spending with consequential rises in domestic prices, imports and (to maintain monetary balance) domestic interest rates. Unlike the unified floating rate case there is no possibility of a fall in the commercial exchange rate to offset the deterioration in the terms of trade, so there is an unambiguous outflow of overseas assets.

It is worth noting that this conclusion, that a dual exchange rate system does not entirely isolate the domestic economy from variations in the overseas interest rate, is at variance with that of earlier authors, notably Cumby (1984). This difference stems entirely from the different assumption made by Cumby about the nature of overseas bonds - that they pay a fixed rate of interest rather than the current rate. On that assumption the fall in face value of overseas bonds which results from a rise in overseas interest rates precisely cancels out the apparent increase in wealth from a rise in the financial rate. So the net effect on domestic spending, prices, income etc. is nil.

b) for periods $t > 0$ again \hat{p} and \hat{r} are assumed to revert to zero, so again rational expectations will place the economy on its unique path which converges back to equilibrium, say with speed λ_1 - the single convergent root of the characteristic equation of the system derived above. So for $t \geq 1$

$$c_t = c_1 \lambda_1^{t-1}$$

so from 4)
$$p_t = \gamma_1^{-1} (1 + r_0 \theta - \lambda_1) c_1 \lambda_1^{t-1}$$

and from 5)
$$r_t = \frac{c_1 (1 + r_0 \theta - \gamma_1 q (1 - \theta) - \lambda_1)}{\alpha \gamma_1} \lambda_1^{t-1}$$

whence from 2)
$$e_t = - \frac{c_1 (1 + r_0 \theta - \gamma_1 q (1 - \theta) - \lambda_1)}{(1 - \lambda_1) \alpha \gamma_1 (1 + r_0)} \lambda_1^{t-1}$$

So our system slides back to equilibrium in exactly the same way as (though in general at a different rate to) what it would do under unified floating rates. Indeed it is noteworthy that the formal expression for the terms of trade at period t is the same under floating and dual rates (although c_1 will differ in the two cases). Both are negative if there has been

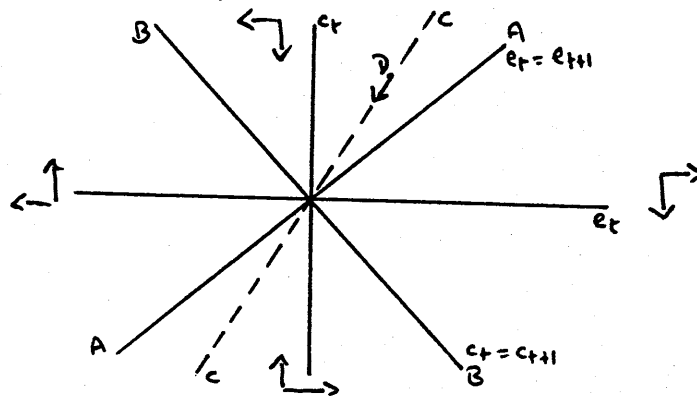
an initial accumulation of overseas assets - and so in both cases these excess assets are gradually disbursed on the path back to equilibrium.

As in the case of unified floating rates we can put our results in the form of a phase diagram. Elimination of p and y from the equations of our model now gives us along the perfect foresight return to equilibrium;

$$A) [q(1-\theta)(\gamma_1 + \beta_2) - s_2]c_t - s_2\beta_1 e_t = -[\alpha(\gamma_1 + \beta_2) + \beta_3](1+r_0)(e_{t+1} - e_t)$$

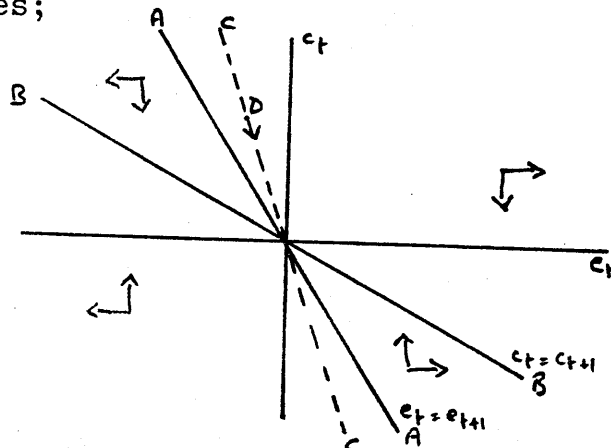
$$B) [q(1-\theta)\gamma_1\beta_3 + \alpha\gamma_1s_2 - r_0\theta(\alpha(\gamma_1 + \beta_2) + \beta_3)]c_t + \alpha\gamma_1s_2\beta_1 e_t = -[\alpha(\gamma_1 + \beta_2) + \beta_3][c_{t+1} - c_t]$$

Now for low values of θ (i.e. $\theta \leq 1 - \frac{s_2}{q(\gamma_1 + \beta_2)}$) provided of course $s_2 < q(\gamma_1 + \beta_2)$ we find the phase diagram;



the stable locus is a line like cc and starting from a point like D the financial exchange rate appreciates as holdings of overseas assets diminish (as originally established by Flood (1977))

As θ increases the slope of AA and BB become steeper (i.e. the gradient of BB becomes increasingly negative). At $\theta = 1 - \frac{s_2}{q(\gamma_1 + \beta_2)}$ the curve AA crosses the $c_t = 0$ axis and our diagram becomes;



So that at high levels of intervention, starting from a point D, and exactly as in the case of floating rates, the financial rate depreciates as surplus overseas assets are disbursed.

As θ continues to grow larger the BB locus continues to swing clockwise and the AA locus to swing anti-clockwise until, when $\theta = \frac{q\gamma_1}{r_0 + q\gamma_1}$ they coincide. This, happily, is the point at which the algebra tells us that a unique rational expectations solution ceases to exist.

As in the case of unified rates it is easy to check that as θ increases so do $\lambda'_1 + \lambda'_2$ and $(1 - \lambda'_1)(1 - \lambda'_2)$ (where λ'_2 is the unstable root of the characteristic equation) whence, arguing exactly as in the case of unified floating rates, as the rate of intervention θ rises so the rate of return to equilibrium λ'_1 gets slower.

Finally for this section we may, as we did for unified rates, calculate the impact of small white noise variations in domestic credit. Equation 5 becomes

$$qm_t + d_t - p_t = \alpha r_t + \gamma r$$

Where d_t is the log exogenous shift in domestic credit in period t . We again assume a single period disturbance at period 0, and find for $t=0$;

$$\begin{aligned} p &= \frac{s_2\beta_1 + (1+r_0)\beta_2}{L} d_0 && \text{positive, so terms of trade are negative} \\ e &= \frac{(\beta_2 + \gamma_1) + \mu(\beta_2 + \gamma_2)}{L} d_0 && \text{positive} \\ r &= - \frac{(1+r_0)[(\beta_2 + \gamma_1) + \mu(\beta_2 + \gamma_2)]}{L} d_0 && \text{negative} \\ c_1 &= - \frac{(\gamma_1 + \mu\gamma_2)[s_2\beta_1 + (1+r_0)\beta_2]}{L} d_0 && \text{negative} \end{aligned}$$

VII Comparison of Different Exchange Rate Regimes

We now compare fixed, floating and dual exchange rate regimes from the point of view of their ability to insulate the domestic economy from the impact of exogenous (and therefore uncontrollable) disturbances. We use as our measure of the efficacy of each regime in this respect the impact on domestic income y of external disturbances. By the Lucas-Sargent equation this amounts to viewing as most desirable that exchange rate regime which minimises the divergence under external disturbance of the domestic price level p_t from its previously expected level $E_{t-1}(p_t)$ - which is the same criterion as that used in Flood (1979).

Since dual rates are fixed for trade transactions and float for financial transactions one might naively expect an economy run under them to behave like an economy run under fixed rates in response to variations in the overseas price level, and like an economy run under floating rates in response to variations in the overseas interest rate. Indeed in a static analysis this must be true. But it turns out that the introduction of rational expectations dynamics yields an interestingly different result.

A casual glance at the rather complicated expressions derived above suggests that any comparison between the effectiveness of the three exchange rate regimes will be very dependent on the various parameter values of the economy. As a matter of mathematics this is true, but for economic purposes we are only interested in making the comparison under plausible parameter values, and to do this it is enough to recall our observations above that ϕ_0 is likely to be relatively

small, and that α and β_2 will be $O(r_0^{-1})$ while the various other parameters appearing in the equations will in general be $O(1)$. In order to make this idea analytically useful we will say that expressions A and B are of the same order of magnitude if $A/B = O(1)$, A is of a lower order of magnitude than B if

$A/B \leq O(r_0)$ and we will write $A \approx B$ if A-B is of a lower order of magnitude than A (this is clearly an equivalence relation). In particular it is easy to see in the expressions above that $K \approx L$.

Thus we can calculate the effect on domestic prices (and so on domestic income) of a given small unanticipated shift in overseas prices under the various exchange rate regimes. Using the results above we find;

$$\frac{\text{impact under floating rates}}{\text{impact under dual rates}} \approx 1$$

$$\frac{\text{impact under fixed rates}}{\text{impact under dual rates}} \approx 1 + \frac{(1+\mu)\beta_2}{\alpha(\beta_2 + \gamma_1 + \mu(\beta_2 + \gamma_2))}$$

Thus we find that, contrary to what naively might seem likely to be the case, dual rates and floating rates are of equal effect (to within $O(r_0)$) in insulating domestic income from variations in the overseas price level, and both are more effective (to an extent dependent on precise parameter values), than fixed rates.

The reasons for this are not far to seek. Both dual and floating rates permit some rise in domestic interest rates (balanced by a shift in exchange rate expectations) to partially offset the increased demand for domestic production brought about

by an overseas price rise. Under floating rates there is also upvaluation in the trade exchange rate but this is small (indeed an order of magnitude small) by comparison. So to within an order of magnitude dual and floating rates have the same effect. Under fixed rates by contrast there can be no offsetting movement in domestic interest rates - so the impact on domestic prices must be that much sharper.

We now turn to the effect on domestic prices of an unanticipated small rise in overseas interest rates. Here we find;

$$\frac{\text{effect under floating rates}}{\text{effect under dual rates}} \approx \frac{\gamma\beta_3 + \alpha(\beta_2 + \delta_1)}{\alpha s_2 \beta_1}$$

and (without writing out the complicated expressions embodying the result) the effect under floating rates and under dual rates is an order of magnitude smaller than the effect under fixed rates.

Looking at the dual rate/floating rate comparison first, it is easy to check (recalling that $\beta_1(s_2 - r_0) = \beta_2 - s_2$ and our assumption (p47) that $r_0 < \frac{s_2 \delta_1}{\beta_2 + \delta_1}$) that the expression above is greater than one, and probably quite considerably so depending upon precise parameter values. Thus dual rates do achieve their intended purpose of more effectively insulating the domestic economy from exogenous capital flows (represented in this model by shifts in r^*). And the way they do it is the obvious one. By decoupling changes in the terms of trade from changes in the financial exchange rate brought about by shifts in overseas interest rates they diminish the impact of such shifts on domestic prices.

The vastly greater effectiveness of both dual and floating rates by comparison with fixed rates in insulating domestic

income from changes in the overseas interest rate is also worthy of note. It turns out that in the case of dual rates the adjustment of exchange rate expectations to a large extent absorbs the shock of any change in r^* , and so diminishes the impact on domestic prices (and income). For floating rates the same effect works (less) but is supplemented by the shift in the terms of trade which further diminishes the impact on domestic prices. Under fixed rates this exchange rate shock absorber is absent, so the impact on domestic prices is greater by an order of magnitude.

We may also look at the impact of exogenous shifts in domestic money supply. As noted above (p45), and as Mundell has famously pointed out, under fixed rates this will have no effect. So the only calculation we need to make is;

$$\frac{\text{effect under floating rates}}{\text{effect under dual rates}} \approx \frac{\beta_2 + \gamma_1 + (1+r_0)\beta_3}{s_2\beta_1 + (1+r_0)\beta_3}$$

by the same reasoning as was used above this expression is greater than one, so dual rates are (to some extent) more effective than floating rates in protecting the domestic economy from this sort of shock too.

Finally for this section we look at the question of the speed of return to equilibrium after an exogenous shock. We confine our comparison to dual rates and floating rates. We will make heavy use of the following (easily proved) lemma; if $u_1 > 0 > u_2$, $v_1 > 0 > v_2$ and $u_1 + u_2 = v_1 + v_2$ then $u_1 u_2 > v_1 v_2$ if and only if $v_1 > u_1$.

We have, from the stability equation under unified floating rates (page 46) that if λ_1 is the stable root of that equation λ_2 its unstable root and $\mu_i = 1 - \lambda_i$ (so $\mu_1 > 0 > \mu_2$);

$$\mu_1 + \mu_2 = \frac{\gamma_1 (s_2 \alpha + (1-\theta) q \beta_3)}{a} - \frac{(1-q)(\beta_2 + \gamma_1)}{(1+r_0)a} - r_0 \theta$$

and

$$\mu_1 \mu_2 = - \frac{(1-q) s_2 \gamma_1}{(1+r_0)a} + \frac{(1-q) r_0 \theta (\beta_2 + \gamma_1)}{(1+r_0)a}$$

a simple computation shows that

$$- \left[-r_0 \theta + \frac{\gamma_1 (s_2 \alpha + (1-\theta) q \beta_3)}{a} \right] \frac{(1-q)(\beta_2 + \gamma_1)}{(1+r_0)a} > - \frac{(1-q) s_2 \gamma_1}{(1+r_0)a} + \frac{(1-q)}{(1+r_0)a} + \frac{(1-q) r_0 \theta (\beta_2 + \gamma_1)}{(1+r_0)a}$$

iff $(1-\theta)q < \frac{s_2}{\beta_2 + \gamma_1}$ (and it is easy to check $\frac{s_2}{\beta_2 + \gamma_1} < 1$)

so by our lemma

$$\mu_1 > \frac{\gamma_1 (s_2 \alpha + (1-\theta) q \beta_3)}{a} - r_0 \theta \quad \text{iff} \quad (1-\theta)q < \frac{s_2}{\beta_2 + \gamma_1}$$

similarly, and with similar notation, for dual rates we have

$$\mu'_1 + \mu'_2 = \frac{\gamma_1 (s_2 \alpha + (1-\theta) q \beta_3)}{a} - \frac{s_2 \beta_1}{(1+r_0)a} - r_0 \theta$$

and

$$\mu'_1 \mu'_2 = - \frac{q \beta_1 s_2 \gamma_1 (1-\theta)}{(1+r_0)a} + \frac{s_2 \beta_1 r_0 \theta}{(1+r_0)a}$$

now

$$- \left[-r_0 \theta + \frac{\gamma_1 (s_2 \alpha + (1-\theta) q \beta_3)}{a} \right] \frac{s_2 \beta_1}{(1+r_0)a} > - \frac{q \beta_1 s_2 \gamma_1 (1-\theta)}{(1+r_0)a} + \frac{s_2 \beta_1 r_0 \theta}{(1+r_0)a}$$

iff $(1-\theta)q > \frac{s_2}{\beta_2 + \gamma_1}$

whence by our lemma

$$\mu'_1 < \frac{\gamma_1 (s_2 \alpha + (1-\theta) q \beta_3)}{a} - r_0 \theta \quad \text{iff} \quad (1-\theta)q > \frac{s_2}{\beta_2 + \gamma_1}$$

It follows that the relationship between the speeds of return to equilibrium under dual and floating rates depends on the rate of intervention, θ , and the proportion, γ , of foreign reserves to total domestic money supply. If the expression $(1-\theta)\gamma$ is high (specifically $(1-\theta)\gamma > \frac{s_2}{\rho_2 + \gamma_1}$) then the rate of return is faster under dual than under unified floating rates. If $(1-\theta)\gamma$ is low then the reverse is true.

The reason for this is not difficult to see. Reverting to our phase diagrams (page 58) we note that the condition $(1-\theta)\gamma > \frac{s_2}{\rho_2 + \gamma_1}$ is precisely the condition for the $e_t = e_{t+1}$ locus to have positive slope. It means that the effect of an exogenously brought about accumulation of overseas assets on the money supply outweighs its effect on prices so that domestic interest rates fall, and hence the financial exchange rate must devalue in expectation of a subsequent appreciation. In the case of unified floating rates this devaluation will necessarily slow the outflow of overseas assets, and hence the return to equilibrium whereas under dual rates (where the commercial exchange rate cannot change) this effect cannot operate.

VIII Dual Rates with no established equilibrium

Hitherto, by making \hat{p}_t^* and \hat{r}_t^* white noise processes about $\hat{p}^* = \hat{r}^* = 0$ we have in effect centred our system on a given equilibrium position (or on a given predictable trend path which has been abstracted out of our small case variables). It is conceivable that this may have unfairly tilted the comparisons made above in favour of those exchange rate systems which contain a fixed component. For where there is

a known equilibrium there is an obvious point at which to set a fixed exchange rate. The problem with moving from mathematics to the real world (or at least one of them) is that the optimal point at which to set your exchange rate becomes less than obvious, and the price you pay for getting it wrong may be greater than just letting it float.

So in this section we extend our comparison of dual and floating rates to the case where p^* and r^* follow an ARIMA (0,1,0) process i.e. abandoning our earlier use of case variables, we assume;

$$r_t^* = R_t + \psi_t^{(1)}$$

where $R_t = R_{t-1} + \eta_t^{(1)}$
and $\psi_t^{(1)}$ and $\eta_t^{(1)}$ are (small) white noise processes.

Similarly, we assume;

$$p_t^* = P_t + \psi_t^{(2)}$$

where $P_t = P_{t-1} + \eta_t^{(2)}$

and $\psi_t^{(2)}$, $\eta_t^{(2)}$ are again white noise processes.

Thus the unobservable variables R_t and P_t follow random walks while the observable variables r_t^* and p_t^* fluctuate around them in a random manner. Such processes are examined e.g. in Chow (1975) whose key result for our present purpose is that if \tilde{R}_t is the best estimate of R_t when r_t^* is known (so that $R_t - \tilde{R}_t$ is a white noise variable) then;

$$\tilde{R}_t = K \tilde{R}_{t-1} + (1-K) r_t^*$$

where

$$K = \frac{\sigma_{\psi_t^{(1)}}^2}{\sigma_{\eta_t^{(1)}}^2 + \sigma_{\psi_t^{(1)}}^2} < 1$$

similarly;

$$\tilde{P}_t = \Lambda \tilde{P}_{t-1} + (1-\Lambda) p_t^*, \text{ with } \Lambda \text{ similarly defined}$$

With r^* and p^* following these processes how should a dual exchange rate system be managed? With a fixed intervention rate θ we have five equations in six unknowns $(y_t, r_t, p_t, e_t^c, e_t^f, c_t)$. Hitherto we have slid round this difficulty by setting $e_t^c = 0$ but under the new regime that is no longer obviously efficient. How should the commercial rate be fixed in those circumstances?

In answering this question we first note a principle which we have implicitly observed in the earlier parts of this chapter: in equilibrium $e_t^c = e_t^f$. This is arbitrary in that it is not imposed by our equations. But it is also sensible; in the real world any enduring differential between the two rates is likely to result in widespread evasion of the regulations on which the dual rate system depends (as the Belgian experience shows cf Decaluwe and Steinherr (1976)). Thus we retain this principle and set e_t^c at that value at which, in equilibrium, we would expect $e_t^c = e_t^f$ (This is not of course the only rule we could adopt. A possible alternative for an Italian, but not Belgian, dual rate system would be to choose that value for e_t^c which would set $E_{t-1}(r_t)$ at some fixed value r' - presumed optimal for, say, growth. The rule adopted here does, however, have the virtue of allowing direct comparison between dual rates and unified exchange rate systems. If dual rates come well out of such a comparison then we know that the authorities can gain by establishing a dual rate system using this rule for setting e_t^c and might do even better using other rules).

In symbols, if we denote by \bar{x}_t the equilibrium value of the variable x when $r^* = r_t^*$, $p^* = p_t^*$ then

$$e_t^c = E_{t-1}(\bar{e}_t^f)$$

With this rule we now proceed to solve our system under dual rates. For any variable x we write

$$x_t = x'_t + E_{t-1}(\bar{x}_t) = x'_t + \hat{x}_t \text{ say}$$

so that \hat{x}_t is the equilibrium value of x_t when $r^* = E_{t-1}(r_t^*) = \hat{r}_t^*$ say and $p^* = E_{t-1}(p_t^*) = \hat{p}_t^*$. Our aim, then, is to split our system of equations into two systems, one in the \hat{x}_t and one in the x'_t .

Equation 1: splits straightforwardly. Clearly $\bar{y}_t = 0$ for all t so

$$(1) \quad \hat{y}_t = 0 \text{ for all } t, \text{ and since } \hat{p}_t = E_{t-1}(\hat{p}_t) \text{ we also have}$$

$$(1') \quad y'_t = \mu(p'_t - E_{t-1}(p'_t))$$

Equation 2: is the complicated one. The key point to note is that from the nature of the processes behind r_t^* and p_t^*

$$E_{t-1}(r_{t+1}^*) = E_{t-1}(R_{t+1}) = E_{t-1}(R_t) = E_{t-1}(r_t^*)$$

$$\text{and } E_{t-1}(p_{t+1}^*) = E_{t-1}(p_t^*)$$

$$\text{so } E_{t-1}(\bar{e}_{t+1}^f) = E_{t-1}(\bar{e}_t^f) = \hat{e}_t^f$$

$$(2) \text{ hence } \hat{r}_t = \hat{r}_t^*$$

and

$$(2') \quad r'_t = r_t^{*'} + (1+r_0)[E_{t-1}(e_{t+1}^{f'}) - e_t^{f'}]$$

Equation 3: In equilibrium, with $r^* = \hat{r}_t^*$, $p^* = \hat{p}_t^*$, $e_t^c = \hat{e}_t^f$ the right hand side vanishes identically to give equation 3 in the \hat{x} . This then leaves;

$$(3) \quad (\hat{c}_{t+1} - \hat{c}_t) + c'_{t+1} - c'_t = \text{RHS of equation 3 with } x'_t \text{ substituted for } x_t^* \text{ throughout, and } e_t^{c'} = 0$$

Equation 4: The same happens as with equation 3. The right hand side, with \hat{x}_t substituted for x_t throughout, vanishes identically to give equation (4) - and leaves

$$4') (\hat{c}_{t+1} - \hat{c}_t) + c'_{t+1} - c'_t = \text{RHS of equation 4 with } x'_t \text{ for } x_t \text{ throughout and } e'_t = 0$$

Equation 5 splits in the obvious way. The \hat{x}_t must satisfy

$$5) q(\hat{e}_t^f + (1-\theta)\hat{c}_t) - \hat{p}_t = -\alpha\hat{r}_t + \hat{y}_t$$

and this leaves

$$5') q(1-\theta)c'_t - p'_t = -\alpha r'_t + y'_t$$

We can now dispose of the \hat{x}_t . The five equations $\hat{1}$ to $\hat{5}$ are simple linear equations in five variables ($\hat{y}_t, \hat{r}_t, \hat{p}_t, \hat{c}_t, \hat{e}_t^f$) depending on two known parameters, $\hat{r}_t^* = E_{t-1}(r_t) = \tilde{r}_{t-1}$ and $p_t^* = \tilde{p}_{t-1}$. They may be solved at each period t and constitute, in a sense, the 'equilibrium' component of the overall solution (i.e. what the solution would be if the system were in equilibrium). It should be noted that since $\hat{y}_t = 0$ movement in this part of the solution does not affect domestic production.

We now split the x'_t part of the solution further. Write

$$x'_t = x_t'' + x_t'''$$

where the x_t''' are solutions of equations 1) to 5) (i.e. our original 5 equations) with $e_t^c = 0$, $r_t^* = r_t^{*'} and $p_t^* = p_t^{*'}$; and the x_t'' are solutions of the equations;$

$$1'') y_t'' = \mu(p_t'' - E_{t-1}(p_t''))$$

$$2'') r_t'' = (1+r_0)[E_{t-1}(e_{t+1}^{f''}) - e_t^{f''}]$$

$$3'') c_{t+1}'' - [1-s_2+r_0\theta]c_t'' = -s_2\beta_1 e_t^{f''} + \beta_2 p_t'' + \beta_2 y_t'' + \beta_2 r_t'' - F_t$$

$$4'') c_{t+1}'' - [1+r_0\theta]c_t'' = -\gamma_1 p_t'' - \gamma_2 y_t'' - F_t$$

$$5'') (1-\theta)c_t'' - (1+q)p_t'' = -\alpha r_t'' + y_t''$$

where F_t the driving function in these equations, is

$$F_t = \hat{c}_{t+1} - \hat{c}_t$$

It is then clear that $x_t'' + x_t'''$ does constitute a solution, and so the solution, to the equations: 1' to 5'.

We look first at the x_t''' . As noted above these satisfy equations 1) - 5) with $e_t^S = 0$ and $r_t^{*'} and p_t^{*}' substituted for r_t^* and p_t^* . Moreover $r_t^{*'} = r_t^* - E_{t-1}(r_t^*)$ and $p_t^{*'} = p_t^* - E_{t-1}(p_t^*)$ are clearly white noise processes (in fact it is easy to check that $r_t^{*'} = \eta_t + \psi_t^{(1)} + (R_{t-1} - \hat{R}_{t-1})$, Hence the x_t''' part of the solution is identical to the solution in the white noise case with zero equilibrium described in section VI, above. This is the part of the overall solution which results from unpredicted changes in the exogenous parameters. In particular the initial impact of unpredicted changes in p^* and r^* are reflected in variations in p''' (and so y''') according to the formulae on p.55 above.$

Finally we look at the x_t'' component of the solution. It is worth noting that the driving function, F_t is again a white noise process. In fact it is straightforward to establish from equations (3) and (4) (using the facts that $\hat{r}_t = \hat{r}_t^*$ and $\hat{y}_t = 0$);

$$\hat{c}_t = \frac{\gamma_1 s_2}{\gamma_1(s_2 - r_0\theta) + \beta_3 r_0\theta} \hat{p}_t^* + \frac{\gamma_1(\beta_2 + \beta_1) + \beta_1\beta_2}{\gamma_1(s_2 - r_0\theta) - \beta_3 r_0\theta} \hat{r}_t^*$$

and

$$\begin{aligned} \hat{p}_{t+1}^* - \hat{p}_t^* &= \tilde{p}_t - \hat{p}_t^* = \lambda \hat{p}_{t+1} + (1-\lambda) p_t^* - \hat{p}_t^* \\ &= (1-\lambda)(p_t^* - \hat{p}_t^*) \\ &= (1-\lambda) p_t^{*'} \end{aligned}$$

similarly

$$\hat{r}_{t+1}^* - \hat{r}_t^* = (1-\kappa) r_t^{*'}$$

so

$$F_t = \hat{c}_{t+1} - \hat{c}_t = \frac{(1-\lambda)\gamma_1 s_2}{\gamma_1(s_2 - r_0\theta) - \beta_3 r_0\theta} p_t^{*'} + \frac{\gamma_1(\beta_2 + \beta_1) + \beta_1\beta_2}{\gamma_1(s_2 - r_0\theta) - \beta_3 r_0\theta} r_t^{*'}$$

Now since the homogeneous form of the equations in the x'' is the same as that for the x''' (and that in Section VI above) it follows that after the initial impact of a shift in the value of the driving function the solution will converge back to zero in the same way, and at the same rate, as in Section VI above. All that remains to be done therefore to complete the solution for the x'' is to compute the initial impact of F_t taking a non-zero value at period t . But casual inspection of the equations under this hypothesis shows that the initial solution must be;

$$\begin{aligned} c_{t+1}'' &= -F_t \\ c_t &= p_t = r_t = e_t^f = y_t = 0 \end{aligned}$$

In particular the x_t'' part of the solution, which might be described as that component which results from modifying expectations as to the likely future course of r^* and p^* does not affect domestic income. The reaction to such a change of expectations is simply to adjust holdings of overseas assets to reflect the new expected equilibrium level.

This completes the solution for dual rates. It is easy to see that exactly the same analysis will go through for unified floating rates (except that now, of course, we need to make no assumption as to how e_t^c is set). Again we can split the solution into three parts; $\hat{x}_t + x_t'' + x_t'''$ and again;

- a) the \hat{x}_t will move as the expected equilibrium if the system moves, and will have no effect on domestic income.
- b) the x_t'' will shift in response to changing expectations as to the future course of r^* and p^* (i.e. in response to a white

noise process). Such shifts will initially be reflected in adjustments to holdings of overseas assets and will then dissipate as the x_t^n tend back to zero at the same rate as in the fixed equilibrium case. Again domestic income will not be affected.

c) the x_t^m reflect the reaction of the system to $r_t^{*'}$ and $p_t^{*'}$ i.e. the unpredicted movements in r^* and p^* . The impact of such movements is of precisely the same magnitude and form as in the fixed equilibrium case.

It follows that a dual rate system is more effective than a unified floating rate system at protecting domestic income from exogenous disturbances even when p^* and r^* follow the processes described above provided e_t^c is set as suggested above.

The extension of this analysis to fixed exchange rates may merit a footnote. Again we need to decide on a strategy for fixing the exchange rate e_t , and again our system is under-specified in that, in equilibrium, we have 5 equations in 6 unknowns ($\bar{y}_t, \bar{u}_t, \bar{m}_t, \bar{p}_t, \bar{r}_t, e_t$). A means of dealing with this compatible with the analysis above is as follows;

a) the Government fixes the mix of overseas assets it wishes the economy to hold in equilibrium (recall that with fixed rates it cannot fix an intervention rate in general) i.e. it fixes an intervention rate θ with

$$\hat{c}_t = \hat{m}_t + \beta \hat{u}_t$$

and

$$\beta \hat{u}_t = \theta \hat{c}_t$$

$$\hat{m}_t = (1-\theta) \hat{c}_t$$

Having thus reduced the number of variables by one the equations unambiguously set a rate \hat{e}_t for e_t which is the rate it would take if allowed to float with $r^* = \hat{r}_t^*$ and $p^* = \hat{p}_t^*$, fix e at this rate for period t . (This is in fact the way the fixed rate

was, implicitly, chosen in the fixed equilibrium version of the model - except that there the result proved to be independent of θ).

With these assumptions the model solves in exactly the same way as before - yielding a solution of the form $\hat{\alpha} + \alpha'' + \alpha'''$ with the $\hat{\alpha}$ reflecting the shifting equilibrium, the α'' driven by $\hat{c}_{t+1} - \hat{c}_t$ and the α''' driven by \hat{p}' and \hat{r}' and both of the latter solving exactly as in the fixed equilibrium case.

IX Conclusion

It may be worth briefly recapitulating our results. We have constructed a simple linear model of an open economy producing a differentiated product in which exchange rates are set by capital movements and balance on the current account is achieved **over time** by the effects of asset movements. We have shown that for such an economy a dual exchange rate system can be expected to be considerably more effective than either fixed or floating rates in insulating domestic economic activity from the effects of external capital movements although even with dual rates the insulation is not total. This is not surprising: it is what the dual exchange rate system was invented for. More interestingly, however, a dual system performs as well as floating rates in insulating domestic production from overseas price rises (where both perform considerably better than fixed rates) and better in insulating it from exogenous variations in domestic money supply.

Moreover these conclusions do not only hold in the easy case when there is an obvious, and fixed, equilibrium value at which to set the commercial exchange rate. Even when our domestic economy lives in a world whose key variables perform a random walk on which we have

/imperfect

imperfect information we are best protected by a dual rate system provided that we adjust the commercial exchange rate sensibly to external conditions - e.g. (as in our model) by setting it to the value we would expect a unified rate to take in equilibrium.

Our model has also cast some light on the effects of Government intervention between the two foreign exchange markets. For low levels of intervention the speed of return to equilibrium after a disturbance is faster in a dual market than under a unified floating rate, and under both systems the exchange rate appreciates as holdings of overseas assets diminish. As the rate of intervention rises the speeds of return diminish, but at differing rates. At a crossover point $(\theta = 1 - \frac{s_2}{q(\eta_1 + \beta_2)})$ the speed of return under unified floating rates becomes faster than under dual rates, although both continue to decrease as θ increases. And the exchange rate now depreciates as holdings of overseas assets diminish. Finally, as θ approaches 1 (a policy of neutral intervention), our model under a Belgian system of dual rates ceases to yield a unique rational expectations solution, suggesting that hitherto unconsidered factors become more prominent in deciding the course of the economy. But under an Italian system our model continues to yield a unique solution up to and including under a policy of neutral intervention.

It may also be worth drawing attention to a couple of possible refinements to our model which might merit further work. The first would be to incorporate some allowance for the 'leakage' between financial and commercial markets which can be expected to take place (and in fact does) whenever a substantial gap opens up between the two rates. This modification would plainly dilute the advantages of the dual rate system as they emerge from our model. The second would be to make our rather rudimentary supply function (equation 1)

more realistic by including in it a term for the effect of the past e.g. by modifying it to

$$y_t = \nu y_{t-1} + \mu (p_t - \bar{E}_{t-1}(p_t))$$

Such a change would yield considerable analytical complications (landing us with a cubic characteristic equation - and hence little prospect of a solution as detailed as that for our present model) and it is not clear what effect it would have on the dual rate/floating rate comparison. But the results we already have (p65) on the speed of return to equilibrium under dual and floating rates is suggestive. If this pattern were maintained then for higher (and, under certain parameter values, all) intervention rates the speed of return of Y to its equilibrium level would be faster under floating rates than under dual rates, whereas at low levels of intervention the reverse would be true. The consequence for the attractiveness of dual rates for any particular economy is likely to depend upon precise parameter values.

Appendix I: Derivation of the Model

We consider small variations of the domestic and world economy about some equilibrium (or, which makes no difference, some predictable trend growth). At the equilibrium domestic interest rates will be equal to overseas interest rates and the exchange rate(s) will be equal to each other and, by choice of units, will take the value 1. As a matter of notation if (upper case) X is any scalar variable then X_0 is its equilibrium value and we define (lower case) x by

$$X = X_0(1+x)$$

so, for small variations about X_0 ; $x \approx \log X/X_0$

We make one exception to this rule, writing for interest rates

$$R = r_0 + r$$

where r_0 , the equilibrium interest rate is assumed small enough not to disequilibrate the model but larger than the small variations from equilibrium r . We now derive the equations of our model as follows;

Equation 1: production

we use the standard Lucas-Sargent equation

$$y_t = \mu (p_t - E_{t-1} p_t)$$

Equation 2: interest rate parity

let E^c be the exchange rate used for current transactions, E^f the rate used for capital transactions (both as units of domestic currency per unit of foreign currency). Let E' denote the rate used for interest earned on foreign bonds (so $E' = E^c$ or E^f depending on the precise rules of the system). Now consider the return at period $t+1$ of S units of domestic currency placed at period t .

If this is placed domestically return is $(1+R)s$

If placed overseas, at interest rate R^* expected return is
(in domestic currency)

$$\frac{s}{E_t^f} [E_{t-1}(E_{t+1}^f)] + \frac{R^*s}{E_t^f} [E_{t-1}(E_{t+1}')]]$$

where, here and henceforth, we adopt the convention that when in period t economic decisions are taken, and the values of the various variables therefore set, the only expectations available to decision takers are those formed on the basis of the information available at the end of period $t-1$.

Capital is assumed infinitely mobile so these two expressions must be equal.

$$(1+r_0+r)s = s \left[\frac{1 + E_{t-1}(e_{t+1}^f)}{1 + e_t^f} + \frac{r_0 + r_t^*}{1 + e_t^f} (1 + E_{t-1}(e_{t+1}')) \right]$$

which, sorted out and using the smallness of e^f and e' gives

$$r_t = r^* - (1+r_0)e_t^f + E_{t-1}(e_{t+1}^f) + r_0 E_{t-1}(e_{t+1}')$$

Equation 3: The Domestic Goods market

We use the identity $X-M = [C+I-S]$ i.e. the trade surplus is equal to net domestic deferred consumption. We add the following assumptions:

a) for wealth effect purposes net wealth is composed entirely of holdings M_t of foreign currency and B_t of foreign bonds (i.e. all but a negligible proportion of real domestic capital is in fact domestically borrowed and so self-cancelling - this is not a necessary assumption but considerably simplifies the analysis).

so

$$W = \frac{E_t^c M_t + E_t^f P_t^* B_t}{P}$$

b) net domestic investment is negatively proportional to domestic interest rates = $T_0 - IR$ say.

c) saving is proportional to income = $s_1 Y$ say, but is offset by a tendency to spend from wealth, $s_2 W$ say, with $s_1, s_2 < 1$.

d) at equilibrium (and again abstracting out any predictable growth trend) net domestic saving and net domestic investment must both be zero (otherwise we have e.g. an indefinitely accumulating pool of domestic investment affecting marginal returns etc.). So in particular $I_0 = I_0$

e) the trade surplus/deficit $X-M$ will be spent on increasing/decreasing holdings of foreign money from M_t to M_{t+1} , and bonds from B_t to B_{t+1} .

putting all this together we find (in foreign currency terms)

$$M_{t+1} - M_t + P_t^* (B_{t+1} - B_t) = R_t^* P_t^* B_t + \frac{P_t}{E_t^c} \left[\frac{-s_2 [E_t^c M_t + E_t^f P_t^* B_t]}{P_t} + s_1 Y_t + I(R_t - r_0) \right]$$

the term $R_t^* P_t^* B_t$ comes from interest on holdings of foreign bonds. Now, dividing through by M_0 and putting

$$\beta_1 = \frac{P_0 B_0}{M_0}, \quad \beta_2 = \frac{s_1 P_0 Y_0}{M_0}, \quad \beta_3 = \frac{I P_0}{M_0}$$

we find substituting $Y_t = Y_0(1+y_t)$ etc;

$$m_{t+1} - m_t + \beta_1 (b_{t+1} - b_t) = [\beta_1 r_0 - s_2 - \beta_1 s_2 + \beta_2] + \beta_1 r^* + \beta_1 r_0 b_t + \beta_1 r_0 P_t^* - s_2 m_t - s_2 \beta_1 (e_t^f - e_t^c + b_t) - s_2 \beta_1 P_t^* + \beta_2 (P_t - e_t^c + y_t) + \beta_3 r_t$$

and for this to balance at equilibrium the term in square brackets must vanish so

$$\beta_2 = s_2 + (s_2 - r_0) \beta_1$$

Equation 4: Demand for exports and imports

We have

change in holdings of external assets = overseas interest earned + receipts from exports - expenditure on imports or, in foreign currency terms.

$$M_{t+1} - M_t + (B_{t+1} - B_t) P^* = R^* P^* B_t + \frac{P X}{E^c} - P^* N$$

where exports (X) are priced at domestic rates, while imports (N) are priced at overseas rates. We now assume

a) demand for imports depends negatively on the commercial terms of trade $\frac{x}{x_0} = \Lambda_c^{u_1}$ where $\Lambda_c = \frac{p^* E^c}{p}$

and $u_1 < 1$

(diminishing marginal overseas demand for exports).

b) demand for imports depends negatively on the commercial terms of trade and positively on domestic income;

$$\frac{N}{N_0} = \Lambda_c^{-u_2} \left(\frac{Y}{Y_0} \right)^{u_3} \text{ and } u_2 < 1$$

(diminishing marginal demand for imports).

Combining these equations, dividing through by M_0 and noting that in equilibrium we must have

$$r_0 p_0 B_0 + p_0 x_0 - p_0 N_0 = 0$$

we find

$$m_{t+1} - m_t + \beta_1 (b_{t+1} - b_t) = \beta_1 r^* + \beta_1 r_0 b_t + \gamma_1 (e_t^c + p_t^* - p_t) - \gamma_2 y_t$$

where $\gamma_2 = \frac{u_3 p_0 N_0}{M_0}$, $\gamma_1 = \frac{p_0 x_0}{M_0} [u_1 + u_2 - 1]$

We assume the Marshall-Lerner condition: $\gamma_1 > 0$. We also assume that equilibrium holdings of exchange are enough to pay for one periods imports

$$\text{so } 1 > \frac{p_0 N_0}{M_0} > \frac{p_0 x_0}{M_0}$$

$$\text{so } \gamma_1 < 1$$

Equation 5: The LM Curve

This is of course standard. The simplest way to get the form we need is to assume that domestic money supply (composed of holdings of foreign money plus a fixed domestic credit component) divided by money G.D.P. is negatively proportional to the domestic interest rate

$$\text{so } \frac{E^c M + D}{pY} = -\alpha R$$

whence $q(e_t^c + m_t) - p_t = -\alpha r_t + y_t$

where $q = \frac{M_0}{M_0 + D} < 1$ and $\alpha = \frac{A}{M_0 + D}$

Appendix III: Italian Type Dual Rates

The only difference to the equations of our model introduced by use of an Italian rather than a Belgian style dual market (i.e. payment of interest on overseas assets through the commercial rather than the financial market) is that the interest rate parity equation becomes;

$$r_t = r_t^* + E_{t-1}(e_{t+1}^f) - (1+r_0)e_t^f + r_0 E_{t-1}(e_{t+1}^c)$$

The major difference that this makes is in the static analysis of the economy, for at equilibrium

$$r = r^* + r_0(e^c - e^f)$$

which allows the economy to settle at points where $r \neq r^*$

This possibility has been extensively canvassed in the literature (as noted in Chapter I).

From the point of view of the present dynamic analysis of responses to small exogenous disturbances, however, use of an Italian rather than a Belgian type dual market system makes little difference - with one exception. Indeed since the only change in the addition of a term of order r_0 to the interest rate parity equation the formulae for the initial impact of exogenous variations in r^* , p^* , and domestic money supply (page 55) will only be changed by some multiple of r_0 , so that our general assumption about the smallness of r_0 ensures that the major conclusions about their magnitude (relative to that of the comparable expressions under unified rates) will not be affected.

Where there is a significant change is in the stability equation (page 54). Under an Italian system this becomes:

$$ae_{t+2} - [(1+r_0)a + s_2\beta_1 + aE + \alpha\gamma_1 D]e_{t+1} + [(1+r_0)aE + (1+r_0)\alpha\gamma_1 D + Es_2\beta_1]e_t = 0$$

it is possible to show as for a Belgian system that the e_t and e_{t+2}

coefficients are positive and the e_{t+1} coefficient negative. Their sum is

$$(\epsilon - 1) [s_2 \beta_1 + r_0 a] + r_0 \alpha \gamma_1$$

the θ coefficient in this can easily be checked to be positive, so the expression is maximal when $\theta = 1$ but then it is;

$$r_0 [r_0 a - s_2 (\alpha \gamma_1 - \beta_1)]$$

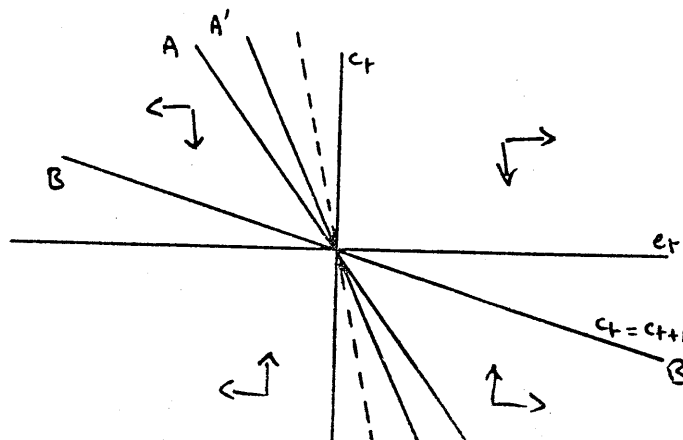
So on the (plausible) assumption that $\beta_1 < \alpha \gamma_1$ and on the continuing assumption that r_0 is small enough not to get in the way (i.e. in this case

$$r_0 < \frac{s_2 (\alpha \gamma_1 - \beta_1)}{\alpha (\beta_2 + \gamma_1) + \beta_3}$$

we find that under an Italian dual market system (unlike under a Belgian dual market system) there is a unique rational expectations solution under a policy of neutral intervention.

Perhaps the best way to see why this should be so is to look again at the phase diagram for higher levels of intervention. The point to note is that the different ways interest is paid under the two systems at any level of the financial exchange rate e_t mean that the rate of domestic interest r_t which will yield identical returns on money placed at home and money placed overseas (and so means $e_{t+1} = e_t$) will be r_t^* under the Belgian system and $r_t^* - r_0 e_t$ under the Italian system. Thus if e.g. $e_t < 0$ the interest rate, and so holdings of overseas assets c_t , will need to be higher under the Italian system than under the Belgian system to ensure $e_t = e_{t+1}$.

Hence in the diagram;



The A'A' locus along which $e_t = e_{t+1}$ under the Italian system is steeper than the AA locus where $e_t = e_{t+1}$ under the Belgian system (it is easy to check that the BB locus is the same under the two systems). Now as noted above as θ increases the AA locus (and the A'A' locus) swing anti-clockwise while the BB locus swings clockwise. When they meet the unique rational expectations solution disappears. But the higher (negative) slope of the A'A' locus ensures that under the Italian system this does not happen for $0 \leq 1$.

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